EC8651 TRANSMISSION LINES AND RF SYSTEMS

UNIT - I

TRANSMISSION LINE THEORY

General theory of Transmission lines - the transmission line - general solution - The infinite line - Wave length, velocity of propagation - Waveform distortion - the distortionless line - Loading and different methods of loading - line not terminated in Z_0 - Reflection co-efficient - Calculation of current, voltage, power delivered and efficiency of transmission - Input and transfer impedance - Open and short circuited lines - reflection factor and reflection loss.

GENERAL THEORY OF TRANSMISSION LINES

Transmission line is a specialized cable or other structure designed to conduct electromagnetic waves in a contained manner.

The lines that carry radio waves from the radio transmitter to the antenna are known as transmission lines.

Transmission line is a conducting structure designed and developed for the transmission of a large amount of electric power in the form of electromagnetic waves from one (place) station to another. They form a connection between the transmitter and receiver to allow the transmission of signals.

Transmission lines are sets of wires, called conductors, that carry electric power from generating plants to the substations that deliver power to customers.

At a generating plant, electric power is stepped up to several thousand volts by a transformer and delivered to the transmission line. If the interconnect length is greater than one fourth of the signal wavelength, transmission line effects become significant and the influence of the interconnect itself must be taken into account.

Types (Based on Length)

1. Short Transmission lines
   - up to 80 Km [upto 50 miles]
2. Medium Transmission lines
   - 80 Km to 240 Km [50 – 150 miles]
3. Long Transmission lines
   - Above 240 Km [> 150 miles]

Types

1. Parallel line
   - ladder line, twisted pair
2. Coaxial cable
   - cable TV line
3. Planar Transmission line
   - strip line / microstrip line

The Transmission Line - General Solution

Transmission line is a conductor or system of conductors that transfers electrical signals from one place to another.

|||ly Transmission line is a conductor designed to carry electricity or an electrical signal over large distances with minimum losses and distortion.

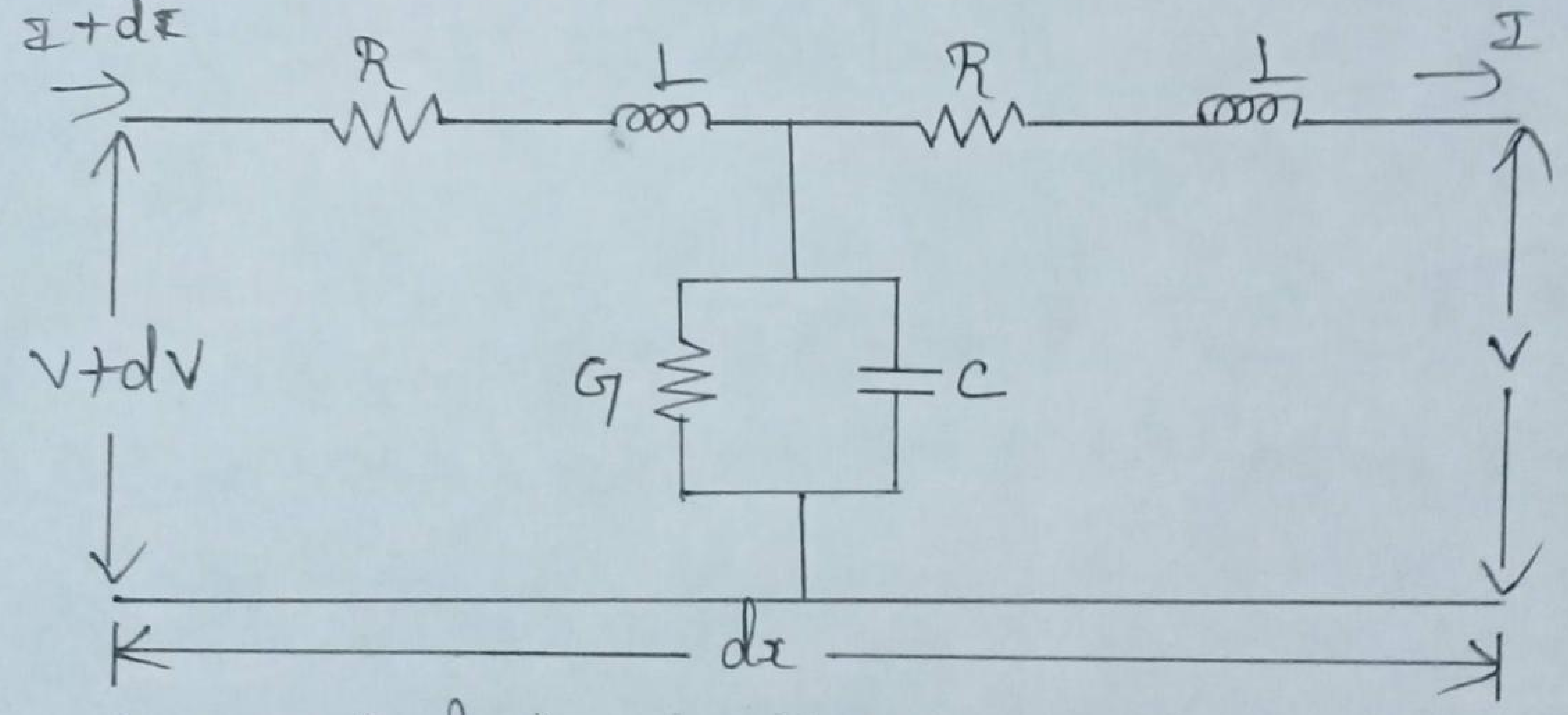

Fig: Equivalent Circuit of Transmission Line [Single 'T' section of length dx]

The series impedance of small 'T' section 'dx' is,

$$Z = (R + j\omega L)\,dx \longrightarrow (1)$$

The shunt admittance of small 'T' section 'dx' is,

$$Y = (G + j\omega C)\,dx \longrightarrow (2)$$

let,

$V + dV$ - sending end voltage

V - receiving end voltage

$I + dI$ - sending end current

I - receiving end current

The potential difference between two ends of 'T' section is,

$$\cancel{V} + dV - \cancel{V} = I(R + j\omega L)dx$$

$$\frac{dV}{dx} = I(R + j\omega L) \rightarrow (3)$$

$$\frac{dV}{dx} = I Z \rightarrow (4) \qquad [\because Z = R + j\omega L]$$

The current difference between two ends of 'T' section is,

$$\cancel{I} + dI - \cancel{I} = V(G + j\omega C)dx$$

$$\frac{dI}{dx} = V(G + j\omega C) \rightarrow (5)$$

$$\frac{dI}{dx} = VY \rightarrow (6) \qquad [\because Y = G + j\omega C]$$

Differentiate equations (4) & (6) with respect to 'x',

$$(4) \Rightarrow \frac{d}{dx}\left(\frac{dV}{dx}\right) = \frac{d(IZ)}{dx}$$

$$\frac{d^2V}{dx^2} = Z\frac{dI}{dx}$$

we know that, $\frac{dI}{dx} = VY$

$$\therefore \frac{d^2V}{dx^2} = Z.VY = VZY$$

we know that, $\gamma = \sqrt{ZY}$

$$\Rightarrow \gamma^2 = ZY$$

$$\therefore \frac{d^2V}{dx^2} = \gamma^2 V \longrightarrow (7)$$

$|||^{ly}$

$$(6) \Rightarrow \quad \frac{d}{dx}\left(\frac{dI}{dx}\right) = \frac{d(\gamma V)}{dx}$$

$$\frac{d^2I}{dx^2} = \gamma \frac{dV}{dx}$$

we know that, $\frac{dV}{dx} = I Z$ [equ.(4)]

$$\therefore \frac{d^2I}{dx^2} = \gamma . I Z = I Z \gamma$$

we know that, $\gamma = \sqrt{Z Y}$

$$\Rightarrow \gamma^2 = Z Y$$

$$\therefore \frac{d^2I}{dx^2} = \gamma^2 I \longrightarrow (8)$$

$$\left.\begin{aligned} (7) &\Rightarrow \frac{d^2V}{dx^2} = \gamma^2 V \\ (8) &\Rightarrow \frac{d^2I}{dx^2} = \gamma^2 I \end{aligned}\right\}$$ These two equations are second order linear differential equations

The assigned solution of the above two equations (7, 8) are,

$$V = Ae^{\gamma x} + Be^{-\gamma x} \longrightarrow (9)$$

$$I = Ce^{\gamma x} + De^{-\gamma x} \longrightarrow (10)$$

where,

A, B, C, D - arbitrary constants

* To find the value of A, B, C & D differentiate equations (9) & (10) with respect to 'x'.

$$(9) \Rightarrow \quad \frac{dV}{dx} = \frac{d}{dx}\left[Ae^{\gamma x} + Be^{-\gamma x}\right] = Ae^{\gamma x} . \gamma + Be^{\gamma x} . -\gamma$$

$$\Rightarrow \frac{dV}{dx} = A\gamma e^{\gamma x} - B\gamma e^{-\gamma x}$$

But $\frac{dV}{dx} = I Z$,

$$\therefore \frac{dV}{dx} = I Z = A\gamma e^{\gamma x} - B\gamma e^{-\gamma x}$$

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$$\Rightarrow \quad ZI = A\sqrt{ZY}\,e^{\gamma x} - B\sqrt{ZY}\,e^{-\gamma x} \qquad [\because \gamma = \sqrt{ZY}]$$

$$I = A\frac{\sqrt{ZY}}{Z}e^{\gamma x} - B\frac{\sqrt{ZY}}{Z}e^{-\gamma x}$$

$$I = A\sqrt{\frac{Y}{Z}}\,e^{\gamma x} - B\sqrt{\frac{Y}{Z}}\,e^{-\gamma x} \longrightarrow (11)$$

$|||^{ly}$

$$(10) \Rightarrow \quad \frac{dI}{dx} = \frac{d\left[Ce^{\gamma x} + De^{-\gamma x}\right]}{dx} = C\gamma e^{\gamma x} - D\gamma e^{-\gamma x}$$

But $\frac{dI}{dx} = VY$

$$\therefore \frac{dI}{dx} = VY = C\gamma e^{\gamma x} - D\gamma e^{-\gamma x}$$

$$\Rightarrow VY = C\sqrt{ZY}\,e^{\gamma x} - D\sqrt{ZY}\,e^{-\gamma x}$$

$$V = \frac{C\sqrt{ZY}}{Y}e^{\gamma x} - \frac{D\sqrt{ZY}}{Y}e^{-\gamma x}$$

$$V = C\sqrt{\frac{Z}{Y}}\,e^{\gamma x} - D\sqrt{\frac{Z}{Y}}\,e^{-\gamma x} \longrightarrow (12)$$

here,

* x – distance / length of the transmission line
* since the distance 'x' is measured from the receiving end of the transmission line,

At the starting point, $x = 0$, $\quad I = I_R$, $V = V_R$, $V_R = I_R Z_R$

where,

I_R – current in the receiving end

V_R – voltage across the receiving end

Z_R – impedance of receiving end

Substitute $x = 0$, $I = I_R$ & $V = V_R$ in equations (9), (10), (11) & (12),

$$(9) \Rightarrow \quad V_R = Ae^{0} + Be^{0} = A + B \longrightarrow (13)$$

$$(10) \Rightarrow \quad I_R = Ce^{0} + De^{0} = C + D \longrightarrow (14)$$

$$(11) \Rightarrow \quad I_R = A\sqrt{\frac{Y}{Z}}e^{0} - B\sqrt{\frac{Y}{Z}}e^{0} = A\sqrt{\frac{Y}{Z}} - B\sqrt{\frac{Y}{Z}} \longrightarrow (15)$$

$$(12) \Rightarrow \quad V_R = C\sqrt{\frac{Z}{Y}}e^{0} - D\sqrt{\frac{Z}{Y}}e^{0} = C\sqrt{\frac{Z}{Y}} - D\sqrt{\frac{Z}{Y}} \longrightarrow (16)$$

To solve equations (13), (14), (15) & (16),

let, $x = \sqrt{\frac{Z}{Y}}$, $\frac{1}{x} = \sqrt{\frac{Y}{Z}}$

now equations (15) & (16) becomes,

$$(15) \Rightarrow \quad I_R = A \cdot \frac{1}{x} - B\frac{1}{x} = \frac{A}{x} - \frac{B}{x} \longrightarrow (17)$$

$$(16) \Rightarrow \quad V_R = Cx - Dx \longrightarrow (18)$$

After solving equations (13), (14), (17) & (18) we get,

$$A = \frac{V_R}{2}\left[1 + \frac{Z_0}{Z_R}\right] \longrightarrow (19)$$

$$B = \frac{V_R}{2}\left[1 - \frac{Z_0}{Z_R}\right] \longrightarrow (20)$$

$$C = \frac{I_R}{2}\left[1 + \frac{Z_R}{Z_0}\right] \longrightarrow (21)$$

$$D = \frac{I_R}{2}\left[1 - \frac{Z_R}{Z_0}\right] \longrightarrow (22)$$

substitute the values of A, B, C & D in equations (9) & (10),

$$V = Ae^{\gamma x} + Be^{-\gamma x} \longrightarrow (9) \qquad I = Ce^{\gamma x} + De^{-\gamma x} \longrightarrow (10)$$

$$(9) \Rightarrow \quad V = \frac{V_R}{2}\left(1 + \frac{Z_0}{Z_R}\right)e^{\gamma x} + \frac{V_R}{2}\left(1 - \frac{Z_0}{Z_R}\right)e^{-\gamma x} \longrightarrow (23)$$

$$(10) \Rightarrow \quad I = \frac{I_R}{2}\left(1 + \frac{Z_R}{Z_0}\right)e^{\gamma x} + \frac{I_R}{2}\left(1 - \frac{Z_R}{Z_0}\right)e^{-\gamma x} \longrightarrow (24)$$

After simplification,

$$(23) \Rightarrow \quad V = \frac{V_R}{2}e^{\gamma x} + \frac{V_R Z_0}{2Z_R}e^{\gamma x} + \frac{V_R}{2}e^{-\gamma x} - \frac{V_R Z_0}{2Z_R}e^{-\gamma x}$$

$$= V_R\left(\frac{e^{\gamma x} + e^{-\gamma x}}{2}\right) + \frac{V_R Z_0}{Z_R}\left(\frac{e^{\gamma x} - e^{-\gamma x}}{2}\right)$$

$$= V_R \cosh \gamma x + \frac{I_R \cancel{Z_R} Z_0}{\cancel{Z_R}} \sinh \gamma x$$

$$V = V_R \cosh \gamma x + I_R Z_0 \sinh \gamma x \longrightarrow (25)$$

$|||^{ly}$

$$(24) \Rightarrow I = \frac{I_R}{2} e^{\gamma x} + \frac{I_R Z_R}{2 Z_0} e^{\gamma x} + \frac{I_R}{2} e^{-\gamma x} - \frac{I_R Z_R}{2 Z_0} e^{-\gamma x}$$

$$= I_R \left(\frac{e^{\gamma x} + e^{-\gamma x}}{2} \right) + \frac{V_R}{Z_0} \left(\frac{e^{\gamma x} - e^{-\gamma x}}{2} \right)$$

$$I = I_R \cosh \gamma x + \frac{V_R}{Z_0} \sinh \gamma x \rightarrow (26)$$

General solution [or voltage and current equations] of a transmission line is,

$$\boxed{\begin{aligned} V &= V_R \cosh \gamma x + I_R Z_0 \sinh \gamma x \\ I &= I_R \cosh \gamma x + \frac{V_R}{Z_0} \sinh \gamma x \end{aligned}}$$

Trignometric Formulae

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}, \quad \sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$\cosh\theta = \frac{e^{\theta} + e^{-\theta}}{2}, \quad \sinh\theta = \frac{e^{\theta} - e^{-\theta}}{2}$$

$$\cos x = \frac{e^{jx} + e^{-jx}}{2}, \quad \sin x = \frac{e^{jx} - e^{-jx}}{2j}$$

$$\cosh x = \frac{e^{x} + e^{-x}}{2}, \quad \sinh x = \frac{e^{x} - e^{-x}}{2}$$

$$\cosh \gamma x = \frac{e^{\gamma x} + e^{-\gamma x}}{2}, \quad \sinh \gamma x = \frac{e^{\gamma x} - e^{-\gamma x}}{2}$$

INFINITE LINE

An infinite line is a line in which the length of the transmission line is infinite. A finite line which is terminated in its characteristic impedance, is termed as infinite line.

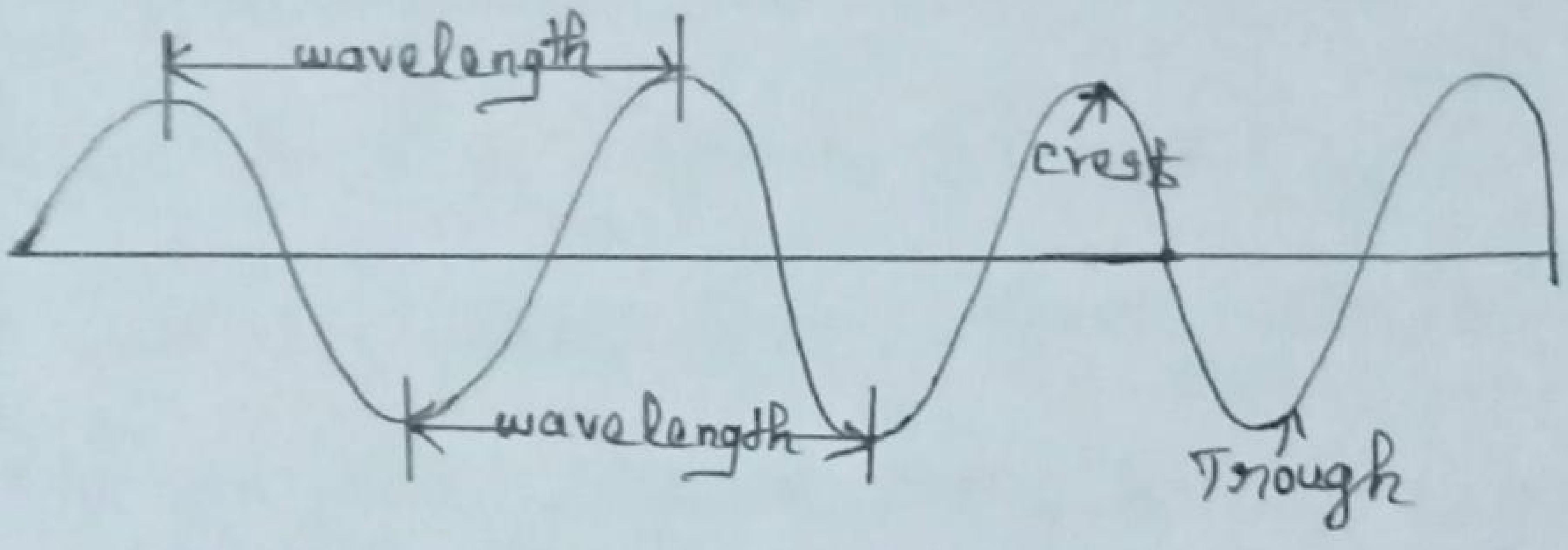
So for an infinite line the input impedance is equivalent to the characteristic impedance. Since the line is of infinite length, no electric wave will ever reach the distant end. Then there will be no possibility of reflection of electric energy at the distant end and consequently no reflected wave also.

The concept of infinite line deals with the analysis of the transmission of electric waves along any uniform and symmetrical transmission line in terms of an imaginary line having an electrical constant per unit length.

Wavelength

Wavelength is the distance between identical points [adjacent crests/adjacent Trough] in the adjacent cycles of a waveform signal propagated in space or along a wire.

Wavelength is the distance between corresponding points of two consecutive waves.



The distance travelled by the wave along the line while the phase angle is changing through 2π radians is called wavelength.

$$\lambda = \frac{2\pi}{\beta}$$

(or)

$$\lambda = \frac{v}{f} \text{ (or) } \frac{c}{f}$$

where,

β - phase shift/ phase constant, radians

c - speed/velocity of light, m/sec

$c = 3 \times 10^8$ m/sec

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Velocity of Propagation

Velocity of propagation is a measure of how fast a signal travels over time or the speed of the transmitted signal as compared to the speed of light.

The velocity of propagation of a transmission line is the speed at which an electrical signal can propagate through the transmission line in comparison to the speed of light.

In a transmission line, signal velocity is the reciprocal of the square root of the capacitance-inductance product. $v = \frac{1}{\sqrt{LC}}$

speed - the time rate at which an object is moving along a path

velocity - the rate and direction of an object's movement

* Propagation constant decides the wave will propagate or not through the medium
* Propagation constant is a complex quantity

$$\gamma = \alpha + j\beta \longrightarrow (1)$$

where,

α - attenuation constant, nepers (or) decibels

β - phase shift / constant / angle, radians

But, $\gamma = \sqrt{ZY} \rightarrow (2)$

where,

$$Z = R + j\omega L$$

$$Y = G + j\omega C$$

Compare equations (1) & (2),

$$\alpha + j\beta = \sqrt{ZY}$$

$$\Rightarrow (\alpha + j\beta) = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$\Rightarrow$ squaring on both sides.

$$(\alpha + j\beta)^2 = \sqrt{(R + j\omega L)(G + j\omega C)}^2$$

$$\alpha^2 + 2j\alpha\beta + j^2\beta^2 = (R + j\omega L)(G + j\omega C)$$

$$\alpha^2 - \beta^2 + j2\alpha\beta = (R + j\omega L)(G + j\omega C)$$

$$\alpha^2 - \beta^2 + j2\alpha\beta = RG + j\omega CR + j\omega LG + j^2\omega^2 LC$$

$$(\alpha^2 - \beta^2) + j2\alpha\beta = (RG - \omega^2 LC) + j\omega[LG + RC] \rightarrow (3)$$

Equating the real parts,

$$\alpha^2 - \beta^2 = RG - \omega^2 LC$$

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$$\Rightarrow \quad \alpha^2 = \beta^2 + RG - \omega^2 LC \longrightarrow (4)$$

Equaling the imaginary parts,

$$2\alpha\beta = \omega(LG + RC)$$

Squaring on both sides,

$$(2\alpha\beta)^2 = [\omega(LG + RC)]^2$$

$$4\alpha^2\beta^2 = \omega^2(LG + RC)^2$$

$$\alpha^2\beta^2 = \frac{\omega^2}{4}(LG + RC)^2 \longrightarrow (5)$$

Substitute the value of α^2 in equ. (5)

$$(\beta^2 + RG - \omega^2 LC)\beta^2 = \frac{\omega^2}{4}(LG + RC)^2$$

$$\beta^4 + \beta^2 RG - \beta^2\omega^2 LC - \frac{\omega^2}{4}(LG + RC)^2 = 0$$

$$\beta^4 + \beta^2(RG - \omega^2 LC) - \frac{\omega^2}{4}(LG + RC)^2 = 0 \rightarrow (6)$$

The solution of the equation (6) is,

$$\beta^2 = \frac{-(RG - \omega^2 LC) \pm \sqrt{(RG - \omega^2 LC)^2 + \cancel{4} \times 1 \times \frac{\omega^2}{\cancel{4}}(LG + RC)^2}}{2 \times 1}$$

$$\Rightarrow \beta^2 = \frac{-(RG - \omega^2 LC) \pm \sqrt{(RG - \omega^2 LC)^2 + \omega^2(LG + RC)^2}}{2}$$

$$\Rightarrow \beta^2 = \frac{(\omega^2 LC - RG) \pm \sqrt{(RG-\omega^2 LC)^2 + \omega^2(LG+RC)^2}}{2}$$

By neglecting -ve values,

$$\boxed{\beta = \sqrt{\frac{(\omega^2 LC - RG) + \sqrt{(RG-\omega^2 LC)^2 + \omega^2(LG+RC)^2}}{2}}} \rightarrow (8)$$

From (4),

$$(4) \Rightarrow \alpha^2 = \beta^2 + RG - \omega^2 LC$$

substitute the value of β in equ (4),

$$\alpha^2 = \frac{\omega^2 LC - RG + \sqrt{(RG-\omega^2 LC) + \omega^2(LG+RC)^2}}{2} + RG - \omega^2 LC \rightarrow (9)$$

$$\Rightarrow \alpha^2 = \frac{\omega^2 LC - RG + \sqrt{(RG-\omega^2 LC) + \omega^2(LG+RC)^2} + 2RG - 2\omega^2 LC}{2}$$

$$\Rightarrow \alpha^2 = \frac{(RG-\omega^2 LC) + \sqrt{(RG-\omega^2 LC) + \omega^2(LG+RC)^2}}{2}$$

$$\Rightarrow \boxed{\alpha = \sqrt{\frac{(RG-\omega^2 LC) + \sqrt{(RG-\omega^2 LC) + \omega^2(LG+RC)^2}}{2}}} \rightarrow (10)$$

For perfect (lossless) transmission line,

$R = 0, \; G = 0$

$$(8) \Rightarrow \quad \therefore \beta = \sqrt{\frac{(\omega^2 LC - 0) + \sqrt{(0-\omega^2 LC)^2 + \omega^2(0)^2}}{2}}$$

$$= \sqrt{\frac{\omega^2 LC + \sqrt{(\omega^2 LC)^2}}{2}}$$

$$= \sqrt{\frac{\omega^2 LC + \omega^2 LC}{2}} = \sqrt{\frac{2\omega^2 LC}{2}}$$

$$\boxed{\beta = \omega\sqrt{LC}}$$

velocity of propagation, $v = \lambda f$ $\left[\because \lambda = c/f \Rightarrow c = \lambda f\right]$

$\times^{ly}$ and $\div$de by 2π,

$\Rightarrow v = \frac{2\pi}{2\pi}\lambda f$

$= 2\pi f \cdot \frac{\lambda}{2\pi}$ $\left[\because \beta = \frac{2\pi}{\lambda} \Rightarrow \frac{1}{\beta} = \frac{\lambda}{2\pi}\right]$

$\Rightarrow \boxed{v = \frac{\omega}{\beta}}$

wkt, $\beta = \omega\sqrt{LC}$,

now, $v = \frac{\cancel{\omega}}{\cancel{\omega}\sqrt{LC}}$

velocity of propagation, $\boxed{v = \frac{1}{\sqrt{LC}}}$

WAVEFORM DISTORTION

Distortion is a change of the original signal, whereas noise is an external random signal added to the original signal.

Signal[~~e~~ voice signal] transmitted over a transmission line is normally complex and consists of many frequency components. Such voice signal will not have all frequencies transmitted with equal attenuation and equal time delay, the received signal will not be identical with the input signal at the sending end. This variation is known as distortion. It is also named as waveform distortion or line distortion. Waveform distortion is an unexpected change in the waveforms of current and voltage as they pass through a device.

The distortions occuring in the transmission line are called waveform distortion or line distortion.

Distortionless Line

A distortionless line has no frequency and no delay distortion.

For a distortionless line,

α - should not be a function of frequency

v - should not be a function of frequency

Z_0 - should not be a function of frequency

β - should be a direct function of frequency

Condition

* β must be a direct function of frequency

$$\beta = \sqrt{\frac{(\omega^2LC - RG) + \sqrt{(RG - \omega^2LC)^2 + \omega^2(LG + RC)^2}}{2}}$$

For β to be a direct function of frequency, the term $\left[(RG-\omega^2LC)^2 + \omega^2(LG+RC)^2\right]$ must be equal to $(RG + \omega^2LC)^2$.

ie, $(RG - \omega^2LC)^2 + \omega^2(LG+RC)^2 = (RG + \omega^2LC)^2$

$$\left.\begin{array}{l}\cancel{R^2G^2} - 2RG\omega^2LC + \cancel{\omega^4L^2C^2} + \\ \omega^2L^2G^2 + \cancel{2\omega^2LGRC} + \omega^2R^2C^2\end{array}\right\} = \cancel{R^2G^2} + 2RG\omega^2LC + \cancel{\omega^4L^2C^2}$$

$$\omega^2L^2G^2 + \omega^2R^2C^2 = 2\omega^2RGLC$$

$$\cancel{\omega^2}\left[L^2G^2 + R^2C^2\right] = 2\cancel{\omega^2}RGLC$$

$$\Rightarrow \quad L^2G^2 + R^2C^2 - 2RGLC = 0$$

$$[LG - RC]^2 = 0$$

$$\Rightarrow \quad LG - RC = 0$$

$$LG = RC$$

$$\boxed{\begin{matrix} \frac{G}{C} = \frac{R}{L} \\ \text{(or)} \\ \frac{L}{C} = \frac{R}{G} \end{matrix}}$$

This is the condition for distortionless line.

i) $\underline{\underline{\beta}}$

Propagation constant, $\gamma = \sqrt{ZY}$

$$= \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$= \sqrt{L\left(\frac{R}{L} + j\omega\right) C\left(\frac{G}{C} + j\omega\right)}$$

we know that,

$$\frac{R}{L} = \frac{G}{C}$$

$$\therefore \gamma = \sqrt{LC \left(\frac{R}{L} + j\omega\right)\left(\frac{R}{L} + j\omega\right)}$$

$$\gamma = \sqrt{LC}\left(\frac{R}{L} + j\omega\right)$$

Phase constant, $\beta = \sqrt{\dfrac{(\omega^2 LC - RG) + \sqrt{(RG - \omega^2 LC)^2 + \omega^2(LG + RC)^2}}{2}}$

but, $\left[\omega^2(LG + RC)^2 + (RG - \omega^2 LC)^2\right] = (RG + \omega^2 LC)^2$

now, $$\beta = \sqrt{\frac{(\omega^2 LC - RG) + \sqrt{(RG + \omega^2 LC)^2}}{2}}$$

$$= \sqrt{\frac{\omega^2 LC - \cancel{RG} + \cancel{RG} + \omega^2 LC}{2}}$$

$$= \sqrt{\frac{\cancel{2}\omega^2 LC}{\cancel{2}}}$$

$$\boxed{\beta = \omega\sqrt{LC}}$$ $\rightarrow$ direct function of frequency

ii) α

α - should not be a function of frequency [ie., independant of frequency]

$$\alpha = \sqrt{\frac{(RG - \omega^2 LC) + \sqrt{(RG - \omega^2 LC)^2 + \omega^2 (LG + RC)^2}}{2}}$$

$\Rightarrow$ To make 'α' is independent of frequency, the term $\left[(RG - \omega^2 LC)^2 + \omega^2 (LG + RC)^2\right]$ should be equal to $(RG + \omega^2 LC)^2$.

ie., $$\left[(RG - \omega^2 LC)^2 + \omega^2 (LG + RC)^2\right] = (RG + \omega^2 LC)^2$$

$$\left.\begin{aligned} &\cancel{R^2 G^2} - 2\omega^2 RGLC + \omega^4 L^2 C^2 + \omega^2 L^2 G^2 \\ &\quad + 2\omega^2 LGRC + R^2 C^2 \omega^2 \end{aligned}\right\} = \cancel{R^2 G^2} + 2\omega^2 \cancel{RGLC} + \cancel{\omega^4 L^2 C^2}$$

$$\omega^2 L^2 G^2 - 2\omega^2 LGRC + \omega^2 R^2 C^2 = 0$$

$$\omega^2 \left[L^2 G^2 - 2LGRC + R^2 C^2 \right] = 0$$

$$\Rightarrow \quad (LG - RC)^2 = 0$$

$$LG - RC = 0$$

$$\Rightarrow LG = RC$$

$$\boxed{\frac{L}{C} = \frac{R}{G} \quad (\text{or}) \quad \frac{G}{C} = \frac{R}{L}}$$

This is the condition for distortionless line.

$$\alpha = \sqrt{\frac{(RG - \omega^2 LC) + \sqrt{(RG - \omega^2 LC)^2 + \omega^2 (LG + RC)^2}}{2}}$$

substitute, $\left[(RG - \omega^2 LC)^2 + \omega^2 (LG + RC)^2\right] = (RG + \omega^2 LC)^2$

now, $$\alpha = \sqrt{\frac{(RG - \omega^2 LC) + \sqrt{(RG + \omega^2 LC)^2}}{2}}$$

$$= \sqrt{\frac{RG - \cancel{\omega^2 LC} + RG + \cancel{\omega^2 LC}}{2}} = \sqrt{\frac{\cancel{2} RG}{\cancel{2}}}$$

$\boxed{\alpha = \sqrt{RG}}$ → independent of frequency

iii) v

velocity of propagation, $v = \frac{\omega}{\beta}$

$$= \frac{\cancel{\omega}}{\cancel{\omega}\sqrt{LC}} \quad \left[\because \beta = \omega\sqrt{LC}\right]$$

$\boxed{v = \frac{1}{\sqrt{LC}}}$ → independent of frequency

iv) Z_0

Characteristic Impedance, $Z_0 = \sqrt{\frac{Z}{Y}}$

$$= \sqrt{\frac{(\;\; + j\omega L)}{(G + j\omega C)}}$$

$$= \sqrt{\frac{L\left(\frac{R}{L} + j\omega\right)}{C\left(\frac{G}{C} + j\omega\right)}}$$

we know that, $\frac{R}{L} = \frac{G}{C}$

$$\therefore \quad Z_0 = \sqrt{\frac{L}{C}\,\frac{\cancel{(G/C + j\omega)}}{\cancel{(G/C + j\omega)}}}$$

$$\boxed{Z_0 = \sqrt{\frac{L}{C}}}$$ $\rightarrow$ independent of frequency

LOADING AND DIFFERENT METHODS OF LOADING

Loading is the process of adding load to power transmission lines by installing the necessary equipment, such as the loading coils in series.

It is necessary to increase L/C ratio to achieve distortionless condition in a transmission line. This can be done by increasing the inductance of a transmission line. Increasing inductance by inserting inductance in series with line is termed as loading and such lines are called loaded lines.

The lumped inductors known as loading coils are placed at suitable intervals along the transmission line to increase the effective distributed inductance. The effect of loading can be realised by comparing the unloading of a transmission line in the attenuation versus frequency graph.

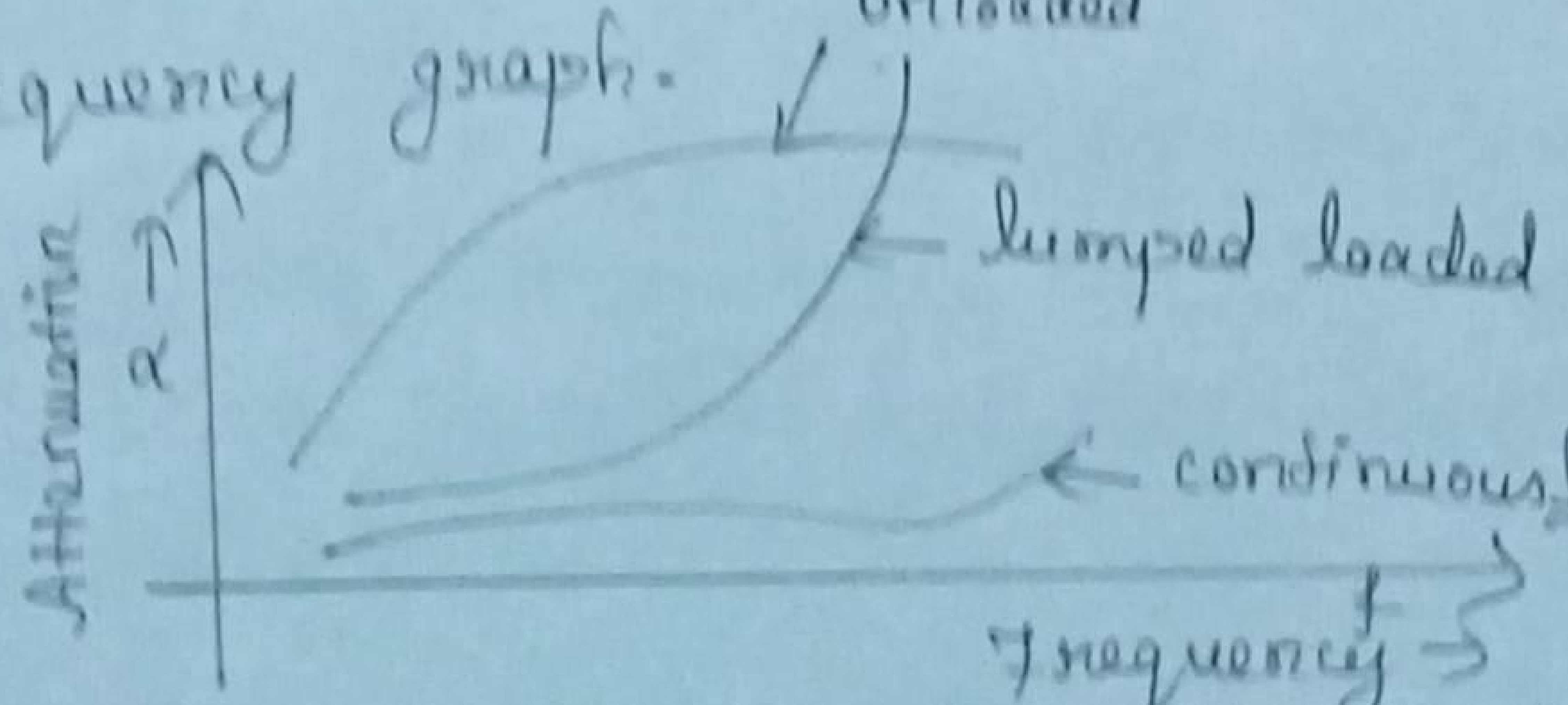

Fig: Comparison of loaded and unloaded cable

Types

1. Lumped Loading
2. Continuous Loading
3. Patch Loading

Lumped Loading

The inductance of a transmission line is increased by the introduction of loading coil at uniform intervals. This is called lumped loading. It acts as a low pass filter. so, it is applicable only for a limited range of frequency.

Continuous Loading

A type of iron or other magnetic material is wound on the transmission line to increase the permeability of the surrounding medium and thereby increase the inductance. This type of loading is called continuous loading.

Patch Loading

Patch loading is the process of using sections of continuously loaded cables separated by sections of unloaded cables which increases the inductance.

Line Not Terminated in Z_0

Consider a transmission line with a voltage source V_s and its impedance Z_s and load impedance Z_R. If Z_R is not equal Z_s and also

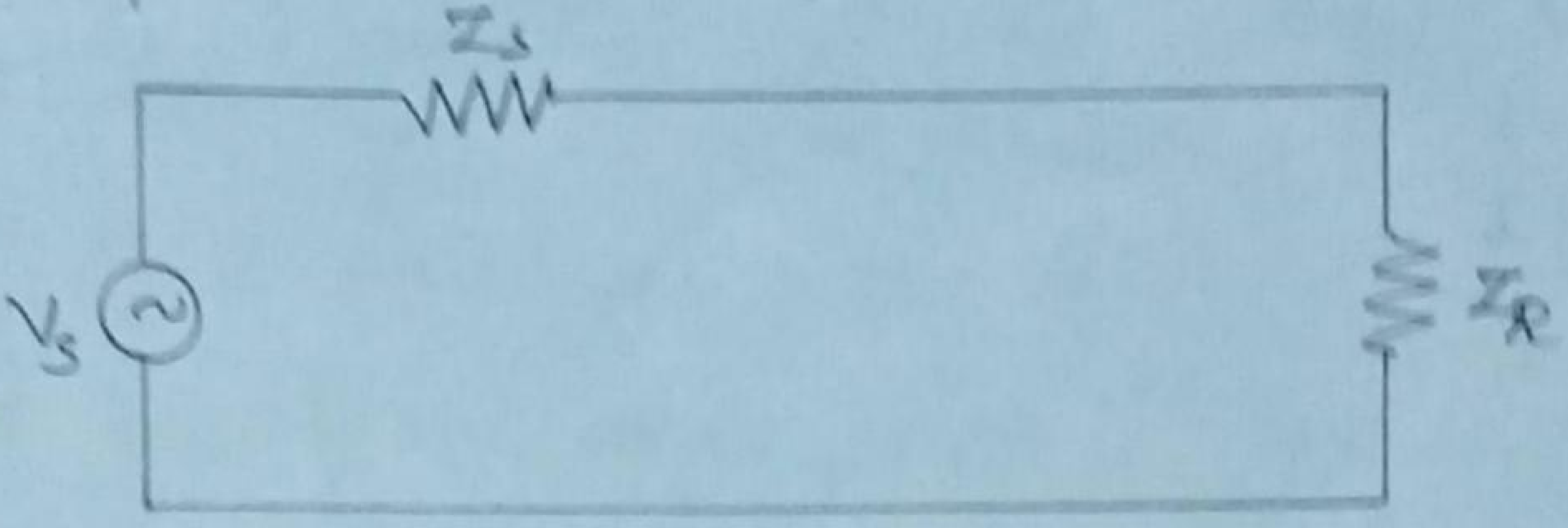

Fig: Transmission line with voltage source V_s and Impedance Z_s

if the line is not terminated with Z_0 reflection takes place. The power delivered to the load is less than that with impedance matching. Reflection results in power loss. This loss is known as reflection loss.

Thus reflection exists for a transmission line which is not terminated in Z_0. Such a reflection is **maximum** when the line is open circuit ($Z_R = \infty$) or short circuit ($Z_R = 0$). The reflection is zero when $Z_R = Z_0$.

From the general solution,

$$V = \frac{V_R(Z_R+Z_0)}{2Z_R} e^{\gamma x} + \frac{V_R(Z_R-Z_0)}{2Z_R} e^{-\gamma x}$$

$$I = \frac{I_R(Z_R+Z_0)}{2Z_0} e^{\gamma x} - \frac{I_R(Z_R-Z_0)}{2Z_0} e^{-\gamma x}$$

* when $Z_R = Z_0 \rightarrow$ no reflection
* only incident wave exist
* the term varying with $e^{\gamma x}$ (+ve 'x') is incident wave
* the term varying with $e^{-\gamma x}$ (−ve 'x') is reflected wave
* when $Z_R \neq 0 \Rightarrow$ reflection takes place
* both incident and reflected waves are exist

$$V = \frac{V_R(Z_R+Z_0)e^{\gamma x}}{2Z_R}$$ – incident voltage wave

$$I = \frac{I_R(Z_R+Z_0)}{2Z_0} e^{\gamma x}$$ – incident current wave

$$V = \frac{V_R(Z_R-Z_0)}{2Z_R} e^{-\gamma x}$$ – reflected voltage wave

$$I = -\frac{I_R(Z_R-Z_0)}{2Z_0} e^{-\gamma x}$$ – reflected current wave

Reflection Co-efficient, $K = \frac{\text{Reflected Voltage}}{\text{Incident Voltage}}$

$$\Rightarrow K = \frac{\frac{1}{Z_R}(Z_R - Z_0)/\cancel{2Z_R}}{\frac{1}{Z_R}(Z_R + Z_0)/\cancel{2Z_R}}$$

$$\boxed{K = \frac{Z_R - Z_0}{Z_R + Z_0}}$$

Reflection factor is the change in current in the load due to reflection at the mismatched junction.

$$\boxed{k = \frac{2\sqrt{Z_S Z_R}}{Z_S + Z_R}}$$

Reflection loss is the reciprocal of reflection factor in nepers or dB.

$$\boxed{\begin{aligned}\text{Reflection loss} &= \ln \frac{1}{k} \\ &= \ln \left| \frac{Z_R + Z_S}{2\sqrt{Z_R + Z_S}} \right| \text{ nepers}\end{aligned}}$$

(or)

$$\boxed{\begin{aligned}\text{Reflection loss} &= 20 \log \frac{1}{k} \\ &= 20 \log \left| \frac{Z_s + Z_R}{2\sqrt{Z_s + Z_R}} \right|\end{aligned}}$$

Calculation of Current, Voltage, Power delivered and Efficiency of Transmission

In general,

Current, $I = \frac{V}{R}$

Voltage, $V = IR$

Power, $P = I^2 R = VI = \frac{V^2}{R}$

Efficiency, $\eta = \frac{\text{Output}}{\text{Input}} \times 100$

For transmission lines,

Sending end Current, $|I_s| = \left| \frac{V_g}{Z_g + Z_s} \right|$

Sending end voltage, $|V_s| = |I_s Z_s|$

Sending end Power, $P_s = |I_s|^2 \cdot R_s$

Receiving end Voltage, $V_R = V_s e^{-\gamma l}$

Receiving end Current, $|I_R| = \left| \frac{V_R}{Z_R} \right|$

Receiving end Power, $P_R = |I_R|^2 \cdot R_R$

Problems

1. An open wire line which is 200km long is properly terminated. The generator at the sending end has V = 10 volts, f = 1 kHz and internal impedance of 500 ohms. At that frequency Z_0 of the line is $(700 - j100)\,\Omega$ and $\gamma = 0.007 + j0.04$ per km. Determine the sending end current, voltage, power and the receiving end current, voltage, power.

Solution

Given, $V_g = 10V$, $Z_g = 500\,\Omega$

$$Z_s = Z_0 = (700 - j100)\,\Omega$$

Sending end current, $|I_s| = \left|\dfrac{V_g}{Z_g + Z_s}\right|$

$$= \left|\frac{10}{500 + 700 - j100}\right|$$

$$= \frac{10}{\sqrt{(1200^2 + 100^2)}}$$

$$= \frac{10}{1204.2}$$

$$|I_s| = 8.3 \times 10^{-3} A = 8.3\,mA$$

Sending end voltage, $|V_s| = |I_s Z_s|$

$$= 8.3 \times 10^{-3} \times \sqrt{700^2 + 100^2}$$

$$= 8.3 \times 10^{-3}$$

$$|V_s| = 5.869 \text{ V}$$

Sending end Power, $P_s = |I_s^2| \cdot R_s$

$$= [8.3 \times 10^{-3}]^2 \times 700$$

$$= 68.89 \times 10^{-6} \times 700$$

$$P_s = 48.22 \text{ mW}$$

Length, $l = 200$ km

Receiving end voltage, $V_R = V_s e^{-\gamma l}$

$$= V_s e^{-(\alpha + j\beta) l}$$

$$= V_s e^{-\alpha l} \cdot e^{-j\beta l}$$

$$= 5.869 \times e^{-0.007 \times 200} \times e^{-j0.04 \times 200}$$

$$= 5.869 \times e^{-1.4} \cdot e^{-j8}$$

$$= 1.45 e^{-j8}$$

$$V_R = [1.45 \angle -8 \text{ radians}] \text{ volts}$$

Magnitude of $V_R = 1.45$, $\beta = -8$ radians

$$\beta = -8 \times \frac{180^\circ}{\underset{3.14}{\pi}} = 458.60^\circ$$

now, $V_R = 1.45 \angle -458.60^\circ$

$V_R = 1.45 \angle -98.60^\circ$ volts

$458.60^\circ \simeq -98.60^\circ$

$458.60 - 360^\circ = 98.60^\circ$

$Z_R = 700 - j100$

$|Z_R| = \sqrt{700^2 - (j100)^2} = \sqrt{700^2 - j^2 100^2}$

$= \sqrt{700^2 + 100^2}$

$Z_R = 707\,\Omega$

Receiving end current, $|I_R| = \left|\frac{V_R}{Z_R}\right|$

$= \frac{1.45}{707}$

$|I_R| = 2.05\,mA$

Receiving end Power, $P_R = |I_R^2| \cdot R_R$

$= (2.05 \times 10^{-3})^2 \times 700$

$= 4.2025 \times 10^{-6} \times 700$

$= 2941.75 \times 10^{-6} = 2.94 \times 10^{-3}$

$P_R = 2.94\,mW$

Input and Transfer Impedance

Input Impedance

The input impedance of a transmission line is the impedance seen by any signal entering it. It is caused by the physical dimensions of the transmission line and its downstream circuit elements.

Input impedance is defined as the ratio of the voltage and current at the pair of the input terminals.

The equations for voltage and current at the sending end of a transmission line of length 'l' is,

$$V_s = V_R \cosh \gamma l + I_R Z_0 \sinh \gamma l$$

$$I_s = I_R \cosh \gamma l + \frac{V_R}{Z_0} \sinh \gamma l$$

Input Impedance of a transmission line is,

$$Z_s = \frac{V_s}{I_s}$$

$$= \frac{V_R \cosh \gamma l + I_R Z_0 \sinh \gamma l}{I_R \cosh \gamma l + \frac{V_R}{Z_0} \sinh \gamma l}$$

$$= \frac{V_R \left[\cosh \gamma l + \frac{I_R Z_0}{V_R} \sinh \gamma l\right]}{I_R \left[\cosh \gamma l + \frac{Z_R}{Z_0} \sinh \gamma l\right]}$$

$[\because V_R = I_R Z_R]$

$$= \frac{\cancel{I_R} Z_R \left[\cosh \gamma l + \frac{\cancel{I_R} Z_0}{\cancel{I_R} Z_R} \sinh \gamma l\right]}{\cancel{I_R} \left[\cosh \gamma l + \frac{Z_R}{Z_0} \sinh \gamma l\right]}$$

$$= \frac{Z_R \cosh \gamma l + \cancel{Z_R} \cdot \frac{Z_0}{\cancel{Z_R}} \sinh \gamma l}{\frac{Z_0 \cosh \gamma l + Z_R \sinh \gamma l}{Z_0}}$$

$$\boxed{\text{Input Impedance, } Z_S = Z_0 \frac{[Z_R \cosh \gamma l + Z_0 \sinh \gamma l]}{[Z_0 \cosh \gamma l + Z_R \sinh \gamma l]}}$$

$$\Rightarrow Z_S = Z_0 \frac{\cancel{\cosh \gamma l} \left[Z_R + Z_0 \frac{\sinh \gamma l}{\cosh \gamma l}\right]}{\cancel{\cosh \gamma l} \left[Z_0 + Z_R \frac{\sinh \gamma l}{\cosh \gamma l}\right]}$$

$$\boxed{\text{Input Impedance, } Z_S = Z_0 \left[\frac{Z_R + Z_0 \tanh \gamma l}{Z_0 + Z_R \tanh \gamma l}\right]}$$

In terms of reflection co-efficient, input impedance is,

$$\boxed{Z_s = Z_0 \left[\frac{e^{\gamma l} + K e^{-\gamma l}}{e^{\gamma l} - K e^{-\gamma l}}\right]}$$

Transfer Impedance

Transfer impedance of a transmission line is defined as the ratio of voltage at the sending end (transmitted voltage) to the current at the receiving end (received current).

Transfer Impedance, $Z_T = \dfrac{V_s}{I_R}$

Sending end voltage,

$$V_s = \frac{V_R}{2}\left[\left(1+\frac{Z_0}{Z_R}\right)e^{\gamma l} + \left(1-\frac{Z_0}{Z_R}\right)e^{-\gamma l}\right]$$

$$= \left[\frac{V_R}{2} + \frac{V_R Z_0}{2 Z_R}\right]e^{\gamma l} + \left[\frac{V_R}{2} - \frac{V_R Z_0}{2 Z_R}\right]e^{-\gamma l}$$

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$$= \left[\frac{I_R Z_R}{2} + \frac{I_R \cancel{Z_R} Z_0}{2\cancel{Z_R}}\right] e^{\gamma l} +$$

$$\left[\frac{I_R Z_R}{2} - \frac{I_R \cancel{Z_R} Z_0}{2\cancel{Z_R}}\right] e^{-\gamma l}$$

$$= I_R \left[\frac{Z_R + Z_0}{2}\right] e^{\gamma l} + I_R \left[\frac{Z_R - Z_0}{2}\right] e^{-\gamma l}$$

$$V_s = I_R \left\{\left(\frac{Z_R + Z_0}{2}\right) e^{\gamma l} + \left(\frac{Z_R - Z_0}{2}\right) e^{-\gamma l}\right\}$$

$$= I_R \left\{\frac{Z_R}{2} e^{\gamma l} + \frac{Z_0}{2} e^{\gamma l} + \frac{Z_R}{2} e^{-\gamma l} - \frac{Z_0}{2} e^{-\gamma l}\right\}$$

$$= I_R \left\{Z_R \left(\frac{e^{\gamma l} + e^{-\gamma l}}{2}\right) + Z_0 \left(\frac{e^{\gamma l} - e^{-\gamma l}}{2}\right)\right\}$$

$$V_s = I_R \left[Z_R \cosh \gamma l + Z_0 \sinh \gamma l\right]$$

// $\cosh \theta = \dfrac{e^{\theta} + e^{-\theta}}{2}$, $\sinh \theta = \dfrac{e^{\theta} - e^{-\theta}}{2}$ //

Now, Transfer Impedance $\Big\}$ $Z_T = \dfrac{V_s}{I_R}$

$$= \frac{\cancel{I_R}\left[Z_R \cosh \gamma l + Z_0 \sinh \gamma l\right]}{\cancel{I_R}}$$

$$\boxed{Z_T = Z_R \cosh \gamma l + Z_0 \sinh \gamma l}$$

Open and Short Circuited Lines

Open Circuited Line

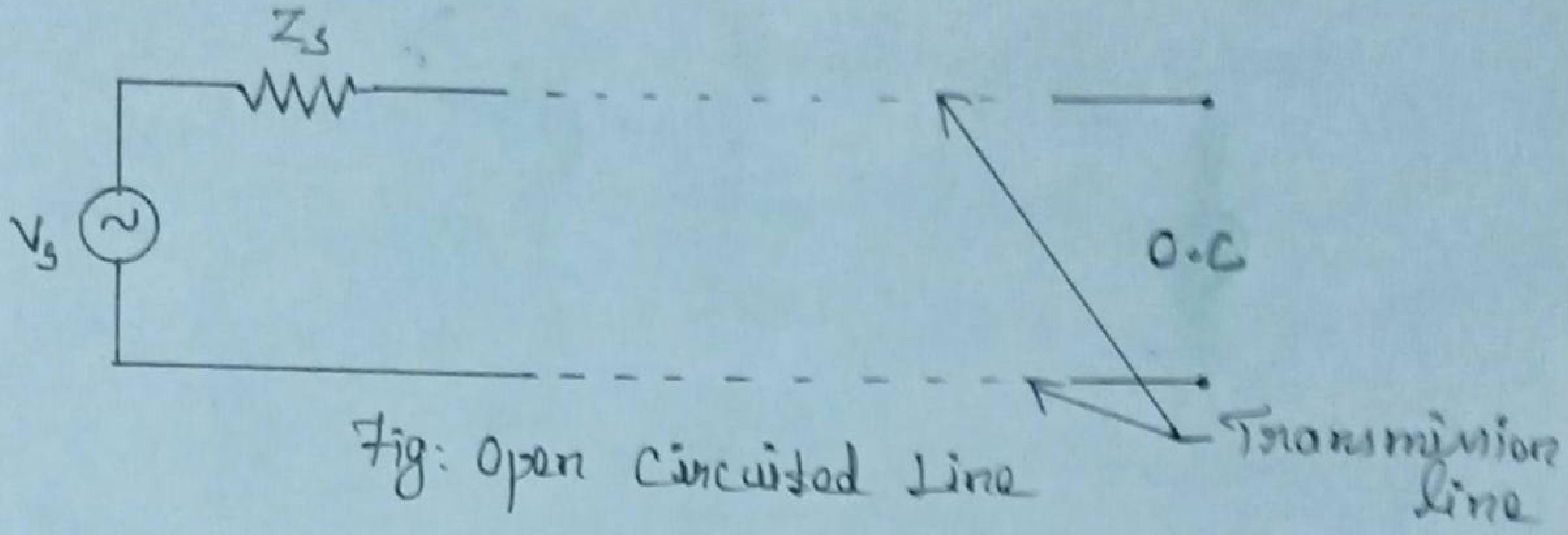

Fig: Open Circuited Line

When the transmission line is opened from the load end, it is known as open circuited transmission line.

* At the open circuited end, the current is minimum and voltage is maximum

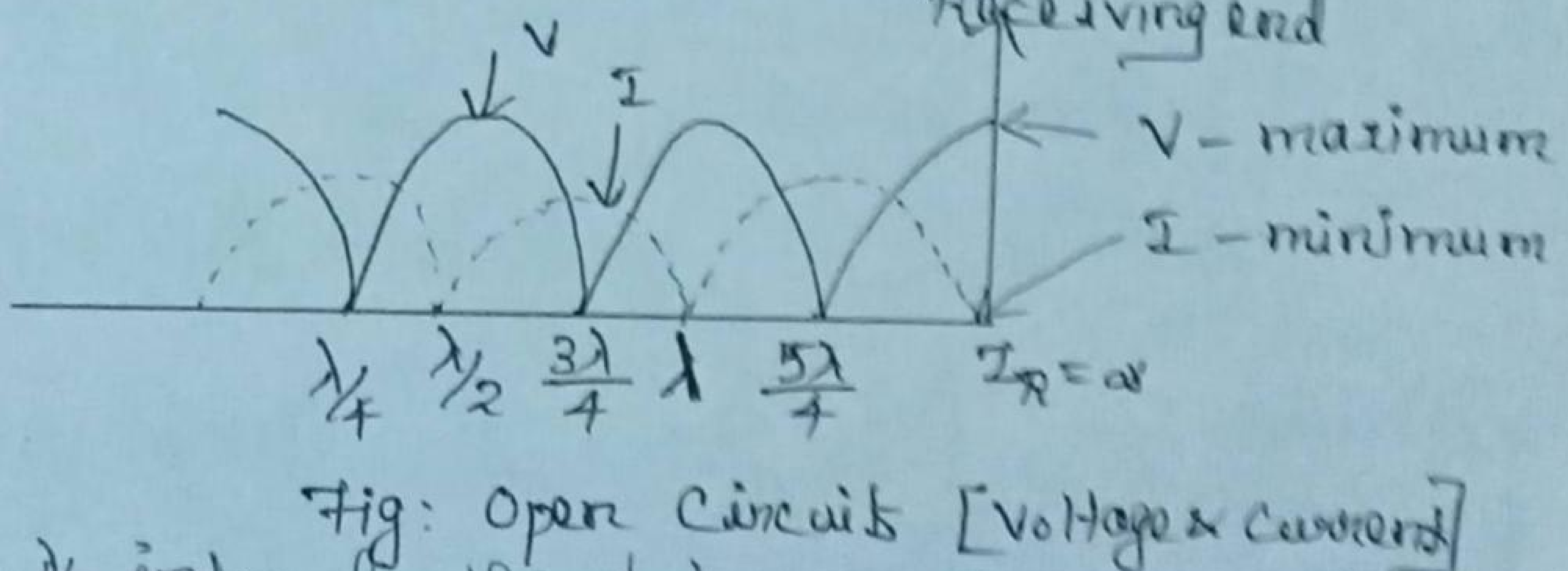

Fig: Open Circuit [Voltage & Current]

* At each $\lambda/2$ interval, this behavior is repeated

The expression for voltage and current of a transmission line is,

$$V = V_R \cosh \gamma l + I_R Z_0 \sinh \gamma l \longrightarrow (1)$$

$$I = I_R \cosh \gamma l + \frac{V_R}{Z_0} \sinh \gamma l \longrightarrow (2)$$

Impedance, $Z = \dfrac{V}{I}$

$$= \frac{V_R \cosh \gamma l + I_R Z_0 \sinh \gamma l}{I_R \cosh \gamma l + \frac{V_R}{Z_0} \sinh \gamma l}$$

replace V_R by $I_R Z_R$

$$Z = \frac{I_R Z_R \cosh \gamma l + I_R Z_0 \sinh \gamma l}{I_R \cosh \gamma l + \frac{I_R Z_R}{Z_0} \sinh \gamma l}$$

$$= \frac{\cancel{I_R} \left[Z_R \cosh \gamma l + Z_0 \sinh \gamma l \right]}{\cancel{I_R} \left[\cosh \gamma l + \frac{Z_R}{Z_0} \sinh \gamma l \right]}$$

$$= \frac{\cancel{\cosh \gamma l} \left[Z_R + Z_0 \frac{\sinh \gamma l}{\cosh \gamma l} \right]}{\cancel{\cosh \gamma l} \left[\frac{Z_0 + Z_R \frac{\sinh \gamma l}{\cosh \gamma l}}{Z_0} \right]}$$

Impedance, $Z = Z_0 \left[\dfrac{Z_R + Z_0 \tanh \gamma l}{Z_0 + Z_R \tanh \gamma l} \right]$

For open circuited line, $Z_R = \infty$

$\therefore$ Impedance, $Z_{oc} = Z_0 \left\{ \dfrac{\cancel{Z_R} \left[1 + \frac{Z_0}{Z_R} \tanh \gamma l \right]}{\cancel{Z_R} \left[\frac{Z_0}{Z_R} + \tanh \gamma l \right]} \right\}$

$$= Z_0 \left\{ \frac{1 + \frac{Z_0}{\infty} \tanh \gamma l}{\frac{Z_0}{\infty} + \tanh \gamma l} \right\}$$

$$= Z_0 \left[\frac{1}{\tanh \gamma l} \right] \qquad \left[\because \frac{1}{\infty} = 0 \right]$$

Open circuit Impedance: $\boxed{Z_{oc} = Z_0 \coth \gamma l}$

Short Circuited Line

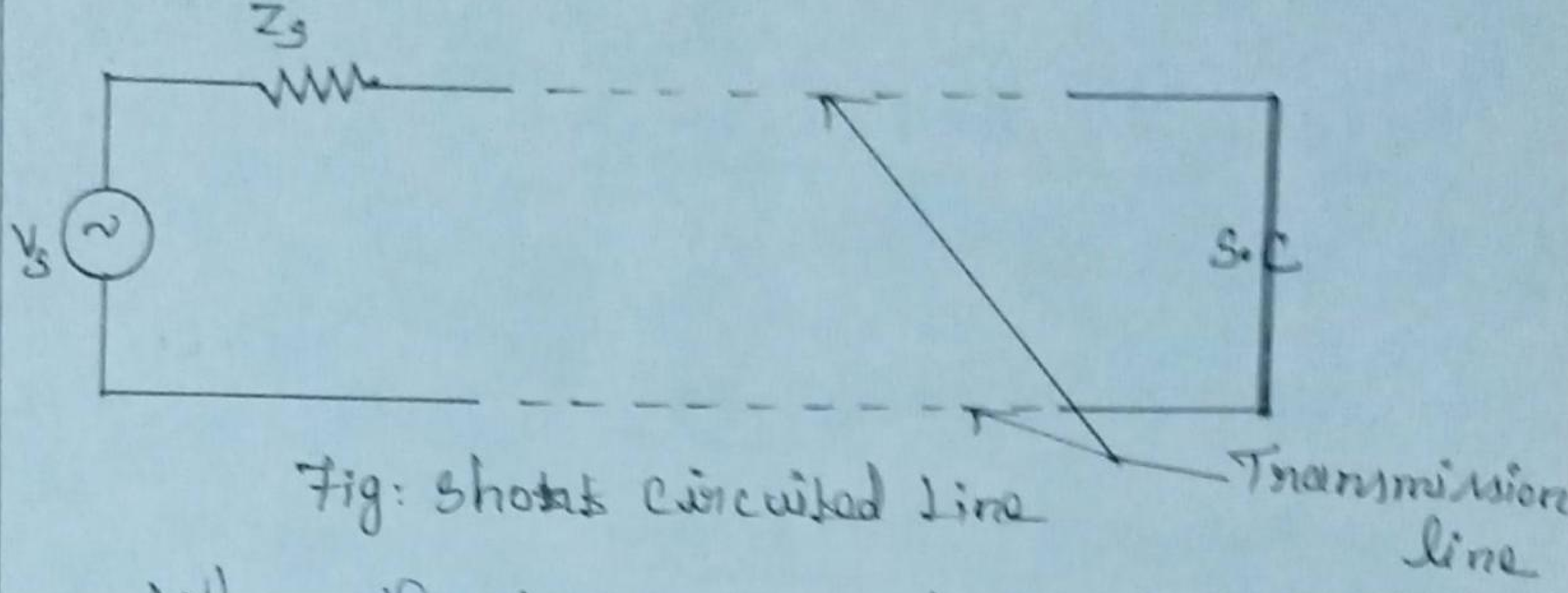

Fig: Short circuited line

When the transmission line is shorted from the load end, it is known as short circuited transmission line.

* At the short circuited end, the current is maximum and voltage is minimum
* At each $\lambda/2$ interval, this behavior is repeated

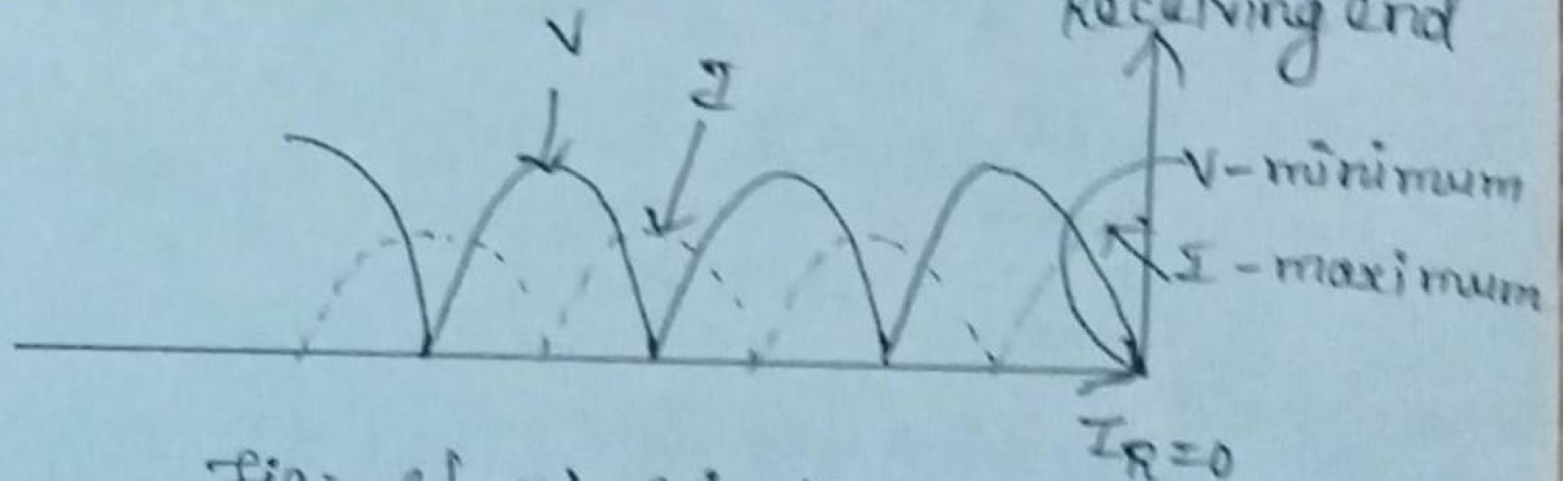

Fig: Short Circuit [voltage and Current]

The expression for voltage and current of a transmission line is,

$$V = V_R \cosh \gamma l + I_R Z_0 \sinh \gamma l \longrightarrow (1)$$

$$I = I_R \cosh \gamma l + \frac{V_R}{Z_0} \sinh \gamma l \longrightarrow (2)$$

Impedance, $Z = \frac{V}{I}$

$$= \frac{V_R \cosh \gamma l + I_R Z_0 \sinh \gamma l}{I_R \cosh \gamma l + \frac{V_R}{Z_0} \sinh \gamma l}$$

$$= \frac{I_R Z_R \cosh \gamma l + I_R Z_0 \sinh \gamma l}{\frac{I_R Z_0 \cosh \gamma l + I_R Z_R \sinh \gamma l}{Z_0}}$$

$$= Z_0 \left\{ \frac{\cancel{I_R \cosh \gamma l} \left[Z_R + Z_0 \tanh \gamma l \right]}{\cancel{I_R \cosh \gamma l} \left[Z_0 + Z_R \tanh \gamma l \right]} \right\}$$

Impedance, $Z = Z_0 \left[\frac{Z_R + Z_0 \tanh \gamma l}{Z_0 + Z_R \tanh \gamma l} \right]$

For short circuited line, $Z_R = 0$

$$\therefore \quad Z_{sc} = Z_0 \left[\frac{0 + Z_0 \tanh \gamma l}{Z_0 + 0} \right]$$

$$= Z_0 \left[\frac{\cancel{Z_0} \tanh \gamma l}{\cancel{Z_0}} \right]$$

Short circuit Impedance, $\boxed{Z_{sc} = Z_0 \tanh \gamma l}$

REFLECTION

A signal travelling along a transmission line will be partly or wholly reflected back in the opposite direction when the travelling signal encounters a discontinuity in the characteristic impedance of the line [or if the far end of the line is not terminated in its characteristic impedance].

ie., When the load impedance is not equal to the characteristic impedance of transmission line reflection occurs.

REFLECTION CO-EFFICIENT

Reflection co-efficient is the ratio of the reflected voltage to the incident voltage at the receiving end of the line.

The voltage equation of transmission line is,

$$V = \frac{V_R(Z_R+Z_0)}{2Z_R} e^{\gamma x} + \frac{V_R(Z_R-Z_0)}{2Z_R} e^{-\gamma x}$$

$\Rightarrow \dfrac{V_R(Z_R+Z_0)}{2Z_R} e^{\gamma x}$ — Incident wave

* the term varying with $e^{\gamma x}$ represents a wave progressing from the sending end towards the receiving end - incidant wave

$$\Rightarrow \frac{V_R(Z_R - Z_0)}{2Z_R} e^{-\gamma x} \text{ - Reflected wave}$$

* the term varying with $e^{-\gamma x}$ represents a wave progressing from the receiving end towards the sending end - reflected wave

$$\text{Reflection co-efficient, } K = \frac{V_R(Z_R - Z_0)/2Z_R}{V_R(Z_R + Z_0)/2Z_R}$$

$$= \frac{\cancel{V_R}(Z_R - Z_0)}{\cancel{V_R}(Z_R + Z_0)} \times \frac{\cancel{2Z_R}}{\cancel{2Z_R}}$$

$$\boxed{K = \frac{Z_R - Z_0}{Z_R + Z_0}}$$

REFLECTION FACTOR

Reflection factor is the ratio of the current actually flowing in the load to that the current might flow under matched condition.

Reflection factor indicates the change in current in the load due to reflection at the mismatched junction.

$$\boxed{k = \left| \frac{2\sqrt{Z_s Z_R}}{Z_s + Z_R} \right|}$$

where,

Z_s - Source [or sending end or input] impedance

Z_R - Load [or receiving end or output] impedance

REFLECTION LOSS

Reflection loss is the reciprocal of the reflection factor in nepers or dB.

$$\text{Reflection loss} = \frac{1}{k}$$

$$\Rightarrow \boxed{\text{Reflection loss} = \ln \left| \frac{Z_S + Z_R}{2\sqrt{Z_S Z_R}} \right| \text{ nepers}}$$

(or)

$$\boxed{\text{Reflection loss} = 20 \log \left| \frac{Z_S + Z_R}{2\sqrt{Z_S Z_R}} \right| \text{ dB}}$$

UNIT - II

HIGH FREQUENCY TRANSMISSION LINES

Transmission line equations at radio frequencies - Line of zero dissipation - Voltage and current on the dissipationless line - Standing waves, Nodes, standing wave ratio - Input impedance of the dissipationless line - Open and short circuited lines - Power and Impedance measurement on lines - Reflection losses - Measurement of VSWR and Wavelength.

TRANSMISSION LINE EQUATIONS AT HIGH FREQUENCIES

Transmission line is a conductor that transfers electrical signals from one place to another.

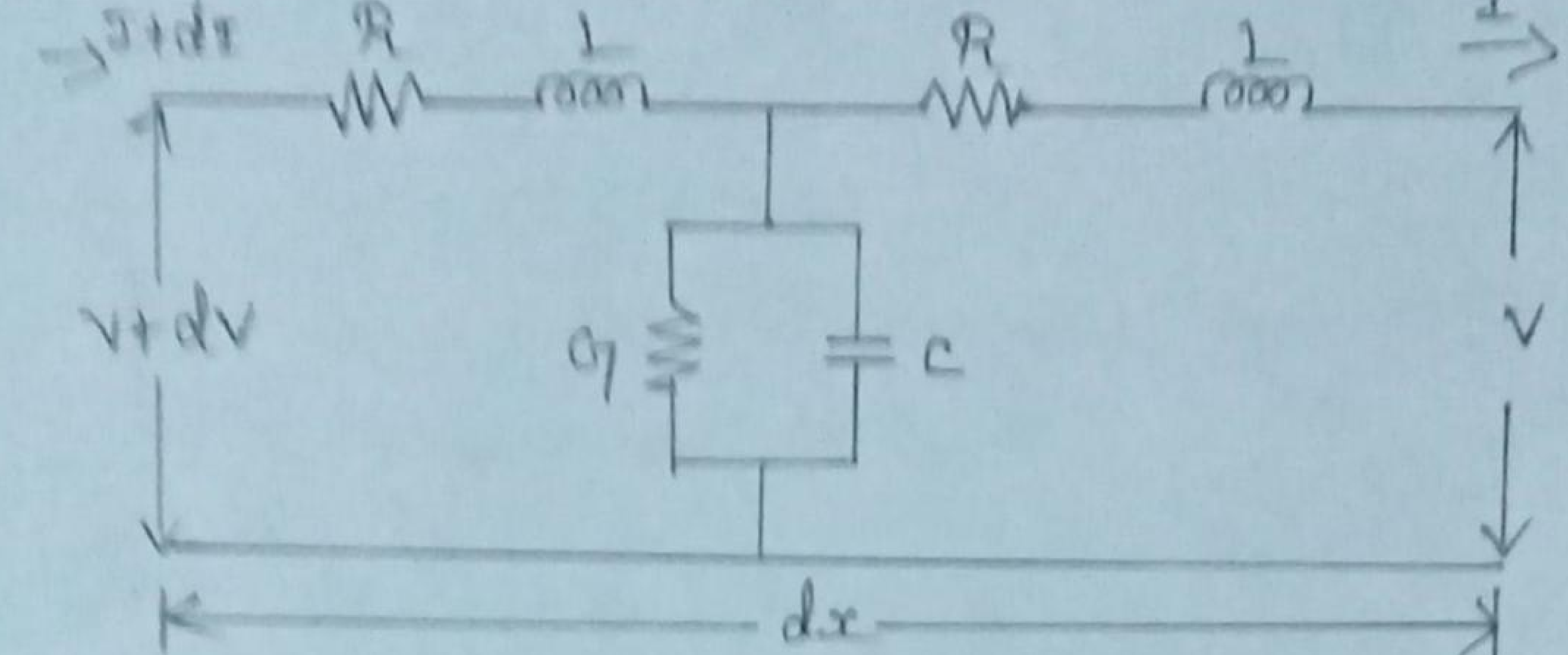

Fig: Equivalent circuit of Transmission Line

Let,

$V+dV$ - voltage at sending end

$I+dI$ - sending end current

V - voltage at receiving end

I - receiving end current

For small 'T' section of length 'dx',

Series impedance, $Z = (R+j\omega L)dx \rightarrow (1)$

Shunt admittance, $Y = (G+j\omega C)dx \rightarrow (2)$

The potential difference between two ends,

$$V+dV-V = I(R+j\omega L)dx$$

$$\Rightarrow \frac{dV}{dx} = I(R+j\omega L) \rightarrow (3)$$

$$\Rightarrow \frac{dV}{dx} = IZ \rightarrow (4)$$

The current difference between two ends,

$$\cancel{I} + dI - \cancel{I} = V(G + j\omega C)\,dx$$

$$\Rightarrow \frac{dI}{dx} = V(G + j\omega C) \longrightarrow (5)$$

$$\Rightarrow \frac{dI}{dx} = VY \longrightarrow (6)$$

After simplification,

⇒ the voltage and current equations of a transmission line is,

$$V = \frac{V_R}{2}\left(\frac{Z_R + Z_0}{Z_R}\right)e^{\gamma x} + \frac{V_R}{2}\left(\frac{Z_R - Z_0}{Z_R}\right)e^{-\gamma x} \longrightarrow (7)$$

$$I = \frac{I_R}{2}\left(\frac{Z_R + Z_0}{Z_0}\right)e^{\gamma x} - \frac{I_R}{2}\left(\frac{Z_R - Z_0}{Z_0}\right)e^{-\gamma x} \longrightarrow (8)$$

High frequency transmission lines are lines, those are designed to carry electromagnetic waves whose wavelengths are shorter than the length of the line.

For high frequency lossless lines,

$$Z_0 = R_0 \,,\quad \gamma = j\beta \,,\quad \alpha = 0$$

now, the general transmission line equations (7) & (8) becomes,

$$V = \frac{V_R}{2Z_R}\left\{(Z_R + R_0)e^{j\beta x} + (Z_R - Z_0)e^{-j\beta x}\right\} \longrightarrow (9)$$

$$I = \frac{I_R}{2R_0}\left\{(Z_R + R_0)e^{j\beta x} - (Z_R - Z_0)e^{-j\beta x}\right\} \longrightarrow (10)$$

From equation (9),

$$(9) \Rightarrow V = \frac{V_R}{2I_R}\left[Z_R e^{j\beta x} + R_0 e^{-j\beta x} + Z_R e^{-j\beta x} - R_0 e^{-j\beta x}\right]$$

$$= \frac{V_R}{2I_R}\left\{Z_R\left(e^{j\beta x} + e^{-j\beta x}\right) + R_0\left(e^{j\beta x} - e^{-j\beta x}\right)\right\}$$

$$= \frac{V_R}{I_R}\left\{Z_R\left(\frac{e^{j\beta x} + e^{-j\beta x}}{2}\right) + R_0\left(\frac{e^{j\beta x} - e^{-j\beta x}}{2}\right)\right\}$$

$$= \frac{V_R}{\cancel{I_R}}\cdot \cancel{I_R} \cos\beta x + j\frac{V_R R_0}{I_R}\sin\beta x$$

$$= V_R\cos\beta x + j\frac{I_R \cancel{Z_R} R_0}{\cancel{Z_R}}\sin\beta x$$

$$V = V_R\cos\beta x + jI_R R_0 \sin\beta x \longrightarrow (11)$$

From equation (10),

$$(10) \Rightarrow I = \frac{I_R}{2R_0}\left[I_R e^{j\beta x} + R_0 e^{-j\beta x} - Z_R e^{-j\beta x} + R_0 e^{-j\beta x}\right]$$

$$= \frac{I_R}{2R_0}\left\{R_0\left(e^{j\beta x} + e^{-j\beta x}\right) + Z_R\left(e^{j\beta x} - e^{-j\beta x}\right)\right\}$$

$$= \frac{I_R}{\cancel{R_0}}\cdot\cancel{R_0}\frac{\left(e^{j\beta x} + e^{-j\beta x}\right)}{2} + \frac{I_R Z_R}{R_0}\frac{\left(e^{j\beta x} - e^{-j\beta x}\right)}{2}$$

$$I = I_R\cos\beta x + j\frac{V_R}{R_0}\sin\beta x \longrightarrow (12)$$

$$\frac{e^{j\beta x} + e^{-j\beta x}}{2} = \cos\beta x, \quad \frac{e^{j\beta x} - e^{-j\beta x}}{2j} = \sin\beta x$$

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∴ The transmission line equations at radio frequencies are,

$$\boxed{\begin{aligned} V &= V_R \cos\beta x + j\, I_R R_0 \sin\beta x \\ I &= I_R \cos\beta x + j\, \frac{V_R}{R_0} \sin\beta x \end{aligned}}$$

Line Constants for Zero Dissipation

Zero Dissipation Line - Dissipationless line

A transmission line is called zero dissipation line/ dissipationless line if the resistance of the line is negligible compare to other parameters of the line.

The line constants for transmission line are,

Resistance (R), Inductance (L),

Capacitance (C) & Conductance (G)

They are related by,

$$Z = R + j\omega L \rightarrow (1)$$

$$Y = G + j\omega C \rightarrow (2)$$

Characteristic Impedance, $Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$ $\rightarrow (3)$

Propagation Constant, $\gamma = \sqrt{ZY} = \sqrt{(R+j\omega L)(G+j\omega C)}$ $\rightarrow (4)$

also, $\gamma = \alpha + j\beta \rightarrow (5)$

For zero dissipation, 'R' is considered as very small when compared to reactance and 'G' is assumed to be zero.

Now, Z & Y becomes,

[$\because$ 'R' is very small $R \approx 0$]

$$Z = j\omega L \rightarrow (6)$$

$$Y = j\omega C \rightarrow (7)$$

now,

Characteristic Impedance $\Bigg\}$ $Z_0 = \sqrt{\frac{Z}{Y}}$

$$= \sqrt{\frac{j\omega L}{j\omega C}}$$

$$\boxed{Z_0 = \sqrt{L/C}}\ \Omega$$

Propagation Constant, $\gamma = \sqrt{ZY}$

$$= \sqrt{j\omega L \times j\omega C}$$

$$= \sqrt{j^2\omega^2 LC}$$

$$\boxed{\gamma = j\omega\sqrt{LC}}$$

But $\gamma = \alpha + j\beta \Rightarrow \gamma = \alpha + j\beta = j\omega\sqrt{LC}$

for zero dissipation line, $\alpha = 0$

$$\therefore \gamma = j\beta = j\omega\sqrt{LC}$$

$$\Rightarrow \boxed{\beta = \omega\sqrt{LC}}$$

Voltage and Current on the dissipationless line

A line with no loss [no power loss] is said to be dissipationless line.

The general expression for voltage and current of the transmission line is,

$$V = \frac{V_R}{2}\left[1 + \frac{Z_0}{Z_R}\right]e^{\gamma x} + \frac{V_R}{2}\left[1 - \frac{Z_0}{Z_R}\right]e^{-\gamma x} \rightarrow (1)$$

$$I = \frac{I_R}{2}\left[1 + \frac{Z_R}{Z_0}\right]e^{\gamma x} + \frac{I_R}{2}\left[1 - \frac{Z_R}{Z_0}\right]e^{-\gamma x} \rightarrow (2)$$

$$(1) \Rightarrow V = \frac{V_R}{2}\left(\frac{Z_R + Z_0}{Z_R}\right)e^{\gamma x} + \frac{V_R}{2}\left(\frac{Z_R - Z_0}{Z_R}\right)e^{-\gamma x}$$

$$V = \frac{V_R}{2}\left(\frac{Z_R + Z_0}{Z_R}\right)\left\{e^{\gamma x} + \left(\frac{Z_R - Z_0}{Z_R + Z_0}\right)e^{-\gamma x}\right\} \rightarrow (3)$$

$$(2) \Rightarrow I = \frac{I_R}{2}\left\{\left(\frac{Z_0 + Z_R}{Z_0}\right)e^{\gamma x} + \frac{I_R}{2}\left(\frac{Z_0 - Z_R}{Z_0}\right)e^{-\gamma x}\right\}$$

$$= \frac{I_R}{2}\left\{\left(\frac{Z_R + Z_0}{Z_0}\right)e^{\gamma x} + \frac{I_R}{2}\left[(-)\frac{(Z_R - Z_0)}{Z_0}\right]e^{-\gamma x}\right\}$$

$$I = \frac{I_R}{2}\left(\frac{Z_R + Z_0}{Z_0}\right)\left\{e^{\gamma x} - \left(\frac{Z_R - Z_0}{Z_R + Z_0}\right)e^{-\gamma x}\right\} \rightarrow (4)$$

For zero dissipation,

$$Z_0 = R_0, \quad \gamma = j\beta \quad [\because \alpha = 0]$$

(3) $\Rightarrow$

$$V = \frac{V_R}{2Z_R}\left\{(Z_R + R_0)e^{j\beta x} + (Z_R - R_0)e^{-j\beta x}\right\}$$

$$V = \frac{V_R}{2Z_R}\left\{Z_R e^{j\beta x} + R_0 e^{j\beta x} + Z_R e^{-j\beta x} - R_0 e^{-j\beta x}\right\}$$

$$= \frac{V_R}{2Z_R}\left\{Z_R\left[e^{j\beta x} + e^{-j\beta x}\right] + R_0\left[e^{j\beta x} - e^{-j\beta x}\right]\right\}$$

$$= \frac{V_R}{Z_R}\left\{Z_R\left[\frac{e^{j\beta x} + e^{-j\beta x}}{2}\right] + R_0\left[\frac{e^{j\beta x} - e^{-j\beta x}}{2}\right]\right\}$$

$$= \frac{V_R}{Z_R}\left[Z_R \cos\beta x + jR_0 \sin\beta x\right]$$

$$= V_R \cos\beta x + j\frac{V_R}{Z_R} R_0 \sin\beta x$$

$$= V_R \cos\beta x + j\frac{I_R \cancel{Z_R}}{\cancel{Z_R}} R_0 \sin\beta x$$

$$\boxed{V = V_R \cos\beta x + j I_R R_0 \sin\beta x}$$

(4) $\Rightarrow$

$$I = \frac{I_R}{2R_0}\left[(Z_R + R_0)e^{j\beta x} - (Z_R - R_0)e^{-j\beta x}\right]$$

$$= \frac{I_R}{2R_0}\left[Z_R e^{j\beta x} + R_0 e^{j\beta x} - Z_R e^{-j\beta x} + R_0 e^{-j\beta x}\right]$$

$$= \frac{I_R}{2R_0}\left\{R_0\left[e^{j\beta x} + e^{-j\beta x}\right] + Z_R\left[e^{j\beta x} - e^{-j\beta x}\right]\right\}$$

$$= \frac{I_R}{R_0}\left\{R_0\left[\frac{e^{j\beta x}+e^{-j\beta x}}{2}\right]+Z_R\left(\frac{e^{j\beta x}-e^{-j\beta x}}{2}\right)\right\}$$

$$= \frac{I_R}{R_0}\left[R_0\cos\beta x + j Z_R \sin\beta x\right]$$

$$= I_R\cos\beta x + j\frac{I_R Z_R}{R_0}\sin\beta x$$

$$\boxed{I = I_R\cos\beta x + j\frac{V_R}{R_0}\sin\beta x}$$

The equations for voltage and current on the dissipationless line are,

$$\boxed{\begin{aligned} V &= V_R\cos\beta x + j I_R R_0 \sin\beta x \\ I &= I_R\cos\beta x + j\frac{V_R}{R_0}\sin\beta x \end{aligned}}$$

Standing Waves

Standing waves are nothing but combination of two waves moving in opposite directions, each having the same amplitude and frequency.

Example

* waves produced by stringed musical instruments — when the string is plucked, pulses travel along the string in opposite directions.

Normally, standing waves are produced whenever two waves of identical frequency interfere with one another while travelling opposite directions along the same medium.

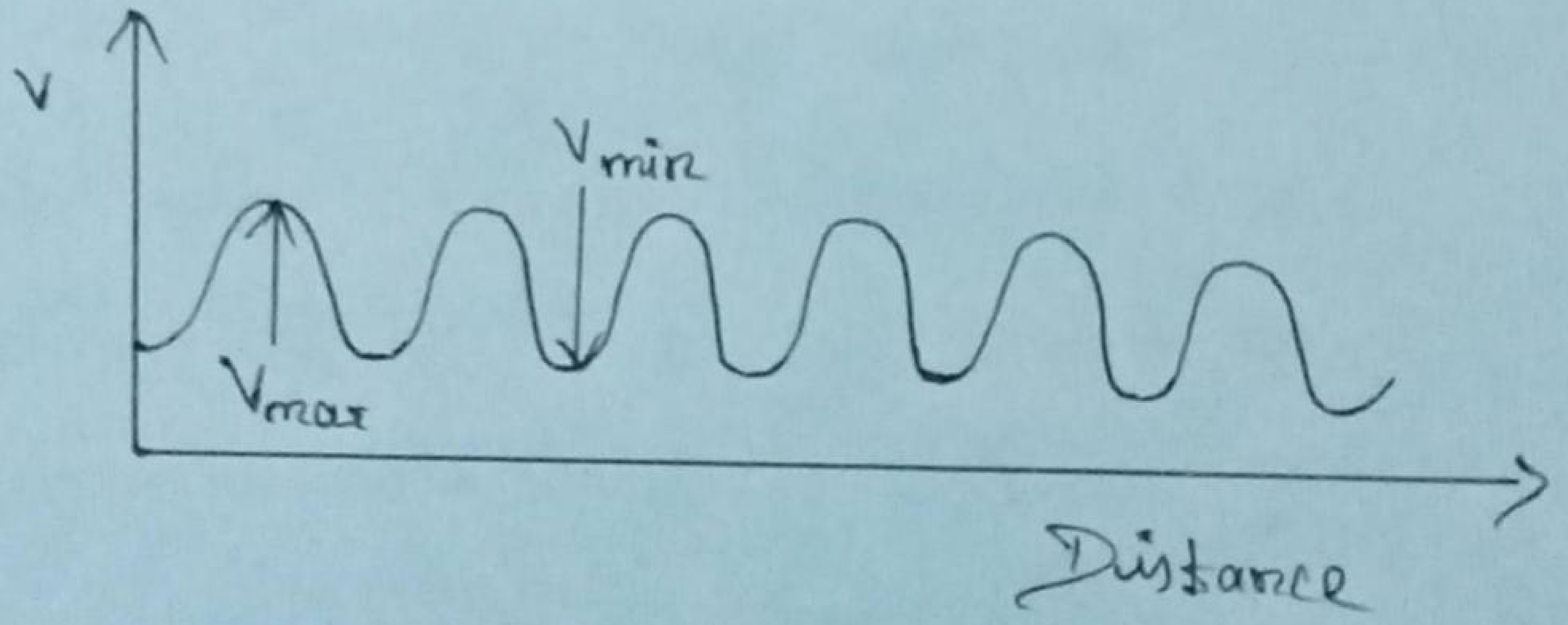
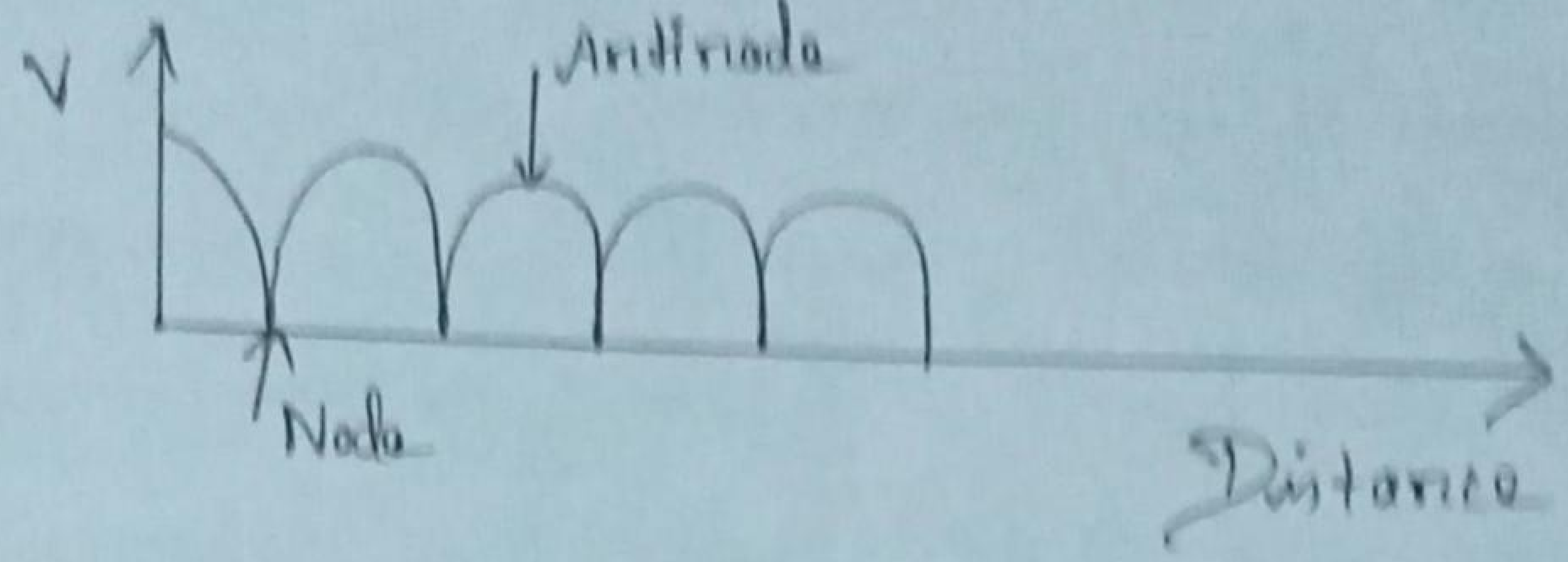


Fig: Standing Wave

A line terminated in its characteristic impedance has no standing waves. It is called a smooth line.

Nodes & Antinodes



Nodes are points of zero voltage or current in the standing waves.

Antinodes are points of maximum voltage or current in the standing waves.

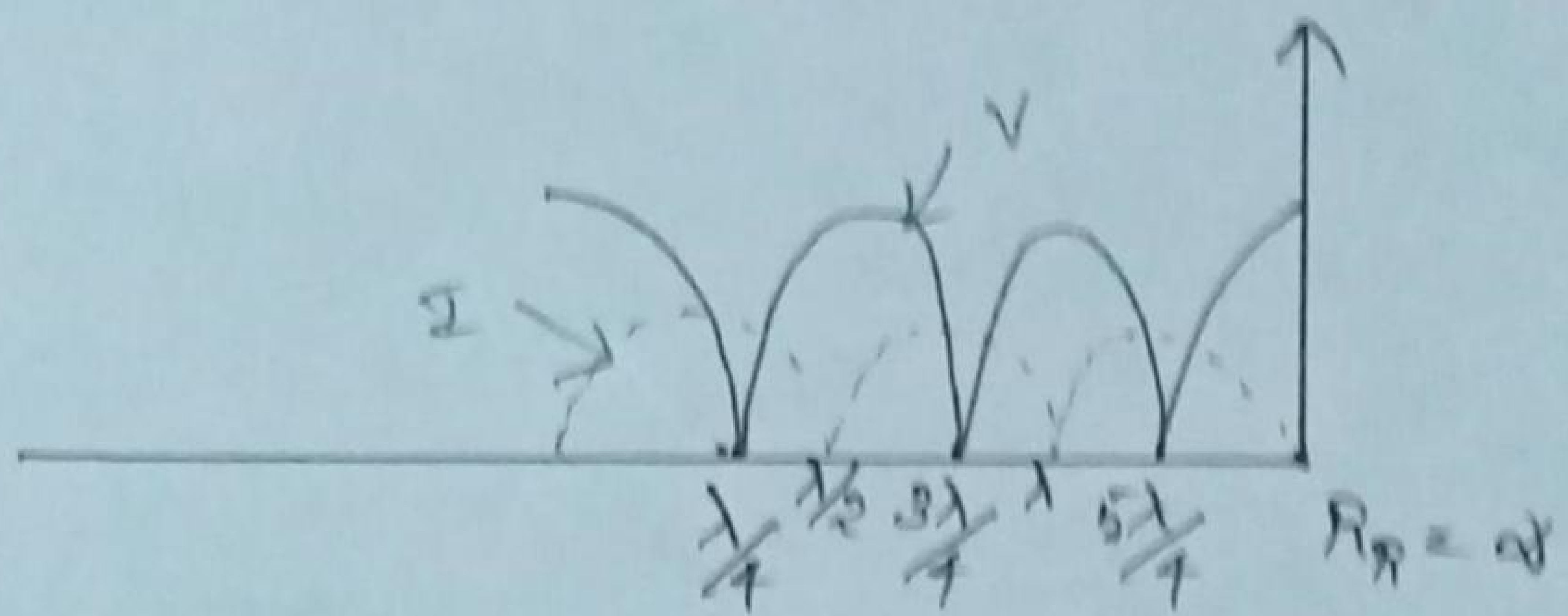


Fig: Open Circuit

For open circuit voltage nodes occur at distances of $\lambda/4$, $3\lambda/4$, $5\lambda/4$ and so on...

Current nodes occur at distances of $0, \lambda/2, \lambda$ and so on...

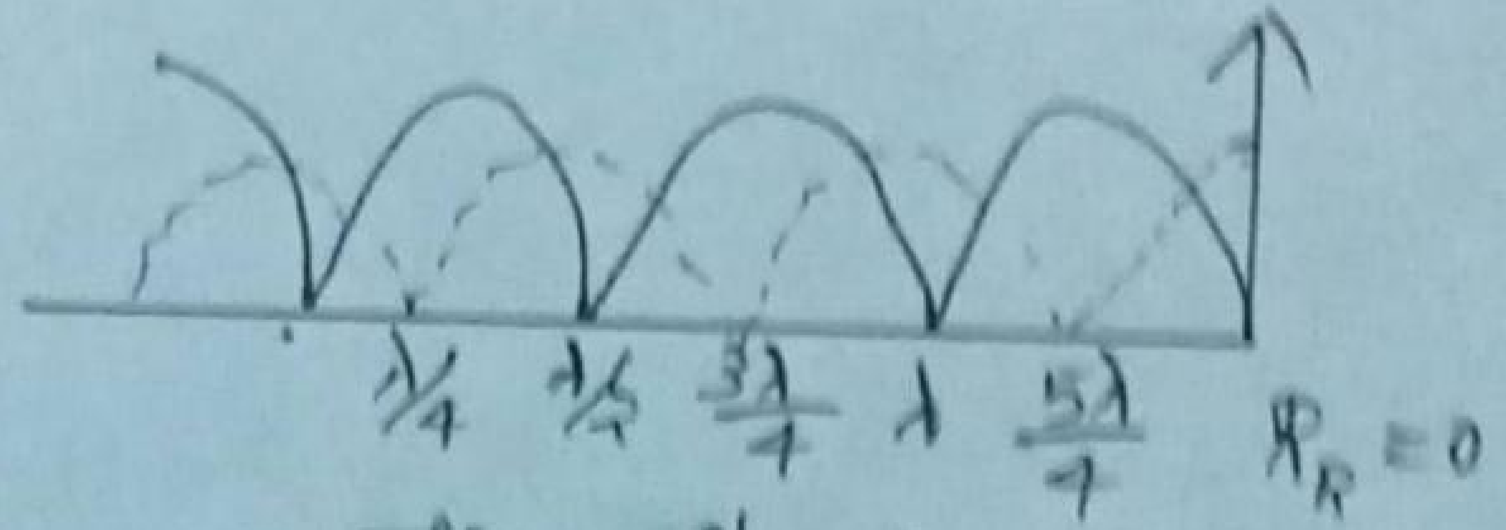


Fig: Short Circuit

For short circuit,
Voltage nodes occur at $0, \lambda/2, \lambda$ .....
Current nodes occur at $\lambda/4, 3\lambda/4, 5\lambda/4$ ......

Standing Wave Ratio

The ratio of the maximum to minimum magnitudes of voltage or current on a line having standing waves is called the standing wave ratio or voltage standing wave ratio.

$$VSWR \text{ (or) } SWR = \left|\frac{V_{max}}{V_{min}}\right| = \left|\frac{I_{max}}{I_{min}}\right|$$

Voltage equation of a transmission line is,

$$V = \frac{V_R(I_R + I_0)}{2I_R}\left[e^{j\beta x} + Ke^{-j\beta x}\right]$$

Incident wave, $$V_{inc} = \frac{V_R(I_R + I_0)}{2I_R} e^{j\beta x}$$

Reflected wave, $$V_{Ref} = \frac{V_R(I_R + I_0)}{2Z_R} e^{-j\beta x}$$

* Maxima of voltage along a line occurs at which the incident and reflected waves are in phase.

$$V_{max} = \frac{V_R(Z_R + Z_0)}{2I_R}\left[1 + |K|\right]$$

* Minima of voltage along a line occurs at which the incident and reflected waves are out of phase.

$$V_{min} = \frac{V_R(I_R + I_0)}{2Z_R}\left[1 - |K|\right]$$

$$\frac{V_{max}}{V_{min}} = \frac{\frac{V_R(\cancel{Z_R+Z_0})}{\cancel{2Z_R}}[1+|K|]}{\frac{V_R(\cancel{Z_R+Z_0})}{\cancel{2Z_R}}[1-|K|]}$$

$$\frac{V_{max}}{V_{min}} = \frac{1+|K|}{1-|K|} = SWR$$

Standing wave ratio in terms of reflection coefficient

$$\boxed{S = \frac{1+|K|}{1-|K|}}$$

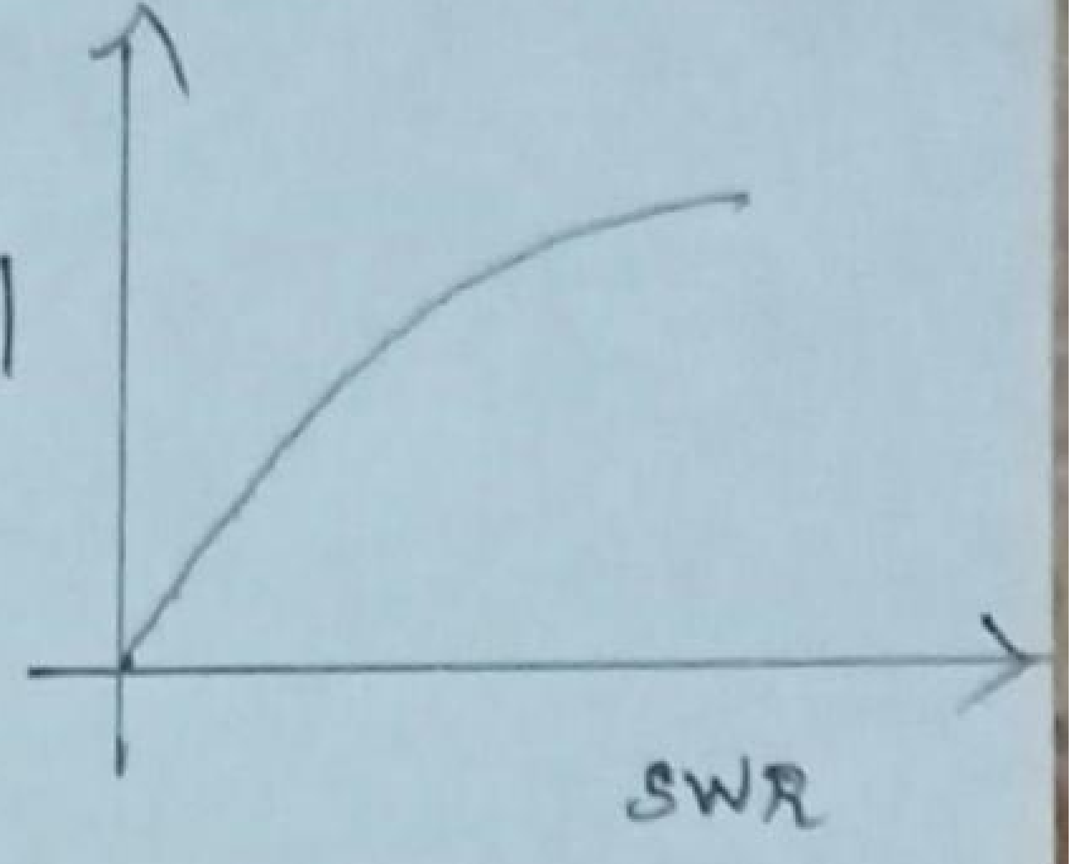


Fig: Relation between SWR and K

$$S(1-|K|) = 1+|K|$$

$$S - S|K| = 1+|K|$$

$$S-1 = S|K| + |K|$$

$$S-1 = |K|[S+1]$$

$$\Rightarrow \boxed{|K| = \frac{S-1}{S+1}}$$

$$|K| = \frac{\left|\frac{V_{max}}{V_{min}}\right| - 1}{\left|\frac{V_{max}}{V_{min}}\right| + 1} = \frac{\frac{V_{max} - V_{min}}{\cancel{V_{min}}}}{\frac{V_{max} + V_{min}}{\cancel{V_{min}}}}$$

$$\boxed{|K| = \frac{|V_{max}| - |V_{min}|}{|V_{max}| + |V_{min}|}}$$

INPUT IMPEDANCE OF THE DISSIPATIONLESS LINE

Transmission line with no power loss is said to be dissipationless line.

Input impedance is the ratio of input voltage to input current.

Input Impedance, $Z_s = \frac{V}{I}$

The voltage and current equation of a lossless transmission line is,

$$V = V_R \cos\beta x + j I_R R_0 \sin\beta x \longrightarrow (1)$$

$$I = I_R \cos\beta x + j \frac{V_R}{R_0} \sin\beta x \longrightarrow (2)$$

now,
$$Z_s = \frac{V}{I} = \frac{V_R \cos\beta x + j I_R R_0 \sin\beta x}{I_R \cos\beta x + j \frac{V_R}{R_0} \sin\beta x}$$

replace, $V_R = I_R Z_R$,

$$\Rightarrow Z_s = \frac{I_R Z_R \cos\beta x + j I_R R_0 \sin\beta x}{I_R \cos\beta x + j \frac{I_R Z_R}{R_0} \sin\beta x}$$

$$= \frac{\cancel{I_R}\left[Z_R \cos\beta x + j R_0 \sin\beta x\right]}{\cancel{I_R}\left[\frac{R_0 \cos\beta x + j Z_R \sin\beta x}{R_0}\right]}$$

Input Impedance
$$\boxed{Z_s = R_0 \frac{Z_R \cos\beta x + j R_0 \sin\beta x}{R_0 \cos\beta x + j Z_R \sin\beta x}}$$

Dividing the numerator and denominator by 'cos βx',

$$Z_s = R_0 \left\{ \frac{\dfrac{Z_R \cos\beta x + j R_0 \sin\beta x}{\cos\beta x}}{\dfrac{R_0 \cos\beta x + j Z_R \sin\beta x}{\cos\beta x}} \right\}$$

$$= R_0 \left\{ \frac{\dfrac{Z_R \cancel{\cos\beta x}}{\cancel{\cos\beta x}} + j \dfrac{R_0 \sin\beta x}{\cos\beta x}}{\dfrac{R_0 \cancel{\cos\beta x}}{\cancel{\cos\beta x}} + j Z_R \dfrac{\sin\beta x}{\cos\beta x}} \right\}$$

Input Impedance

$$\boxed{Z_s = R_0 \left[\frac{Z_R + j R_0 \tan\beta x}{R_0 + j Z_R \tan\beta x} \right]}$$

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Open and Short Circuited Lines

Open Circuited Line

When the transmission line is opened from the load end, it is known as open circuited (transmission) line.

The voltage and current equation of a lossless transmission line (zero dissipation line) is,

$$V = V_R \cos\beta x + j I_R R_0 \sin\beta x$$

$$I = I_R \cos\beta x + j \frac{V_R}{R_0} \sin\beta x$$

Input Impedance, $Z_S = R_0 \left[\frac{Z_R + j R_0 \tan\beta x}{R_0 + j Z_R \tan\beta x} \right]$

For open circuited line, $Z_R = \infty$

replace Z_S by Z_{OC}

Open Circuit Impedance, $$Z_{OC} = R_0 \left\{ \frac{\cancel{Z_R}\left[1 + \frac{j R_0}{Z_R} \tan\beta x\right]}{\cancel{Z_R}\left[\frac{R_0}{Z_R} + j \tan\beta x\right]} \right\}$$

$$= R_0 \left[\frac{1 + \frac{j R_0}{\infty} \tan\beta x}{\frac{R_0}{\infty} + j \tan\beta x} \right]$$

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$$= R_0 \left[\frac{1+0}{0 + j\tan\beta x} \right] \qquad \left[\because 1/\infty = 0\right]$$

$$= R_0 \cdot \frac{1}{j\tan\beta x} \times \frac{j}{j}$$

$$= \frac{j R_0}{j^2 \tan\beta x} = \frac{-j R_0}{\tan\beta x}$$

$$\boxed{Z_{oc} = -j R_0 \cot\beta x} \quad (\text{or}) \quad \boxed{Z_{oc} = \frac{-j R_0}{\tan\beta x}}$$

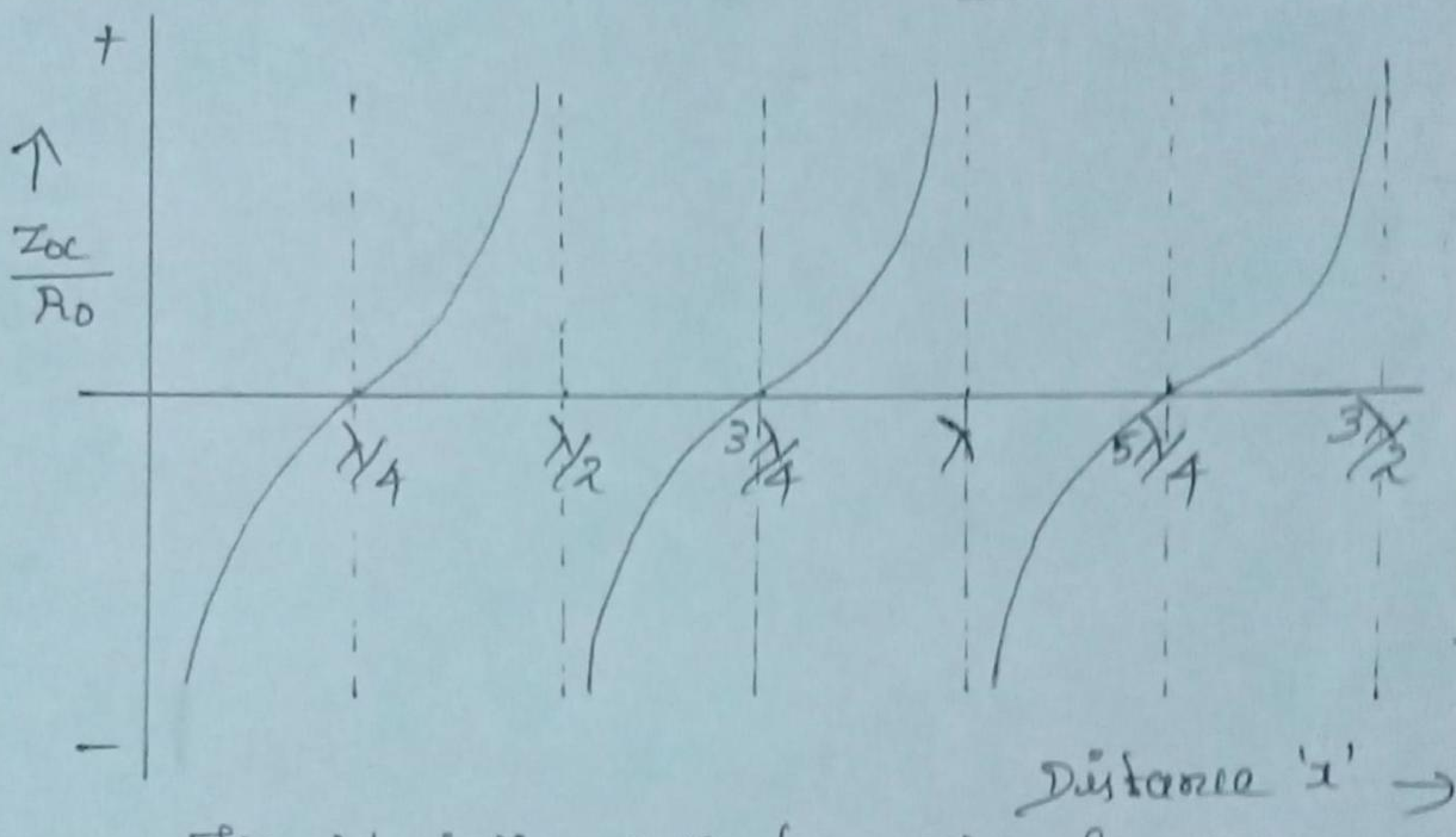

Fig: Variation of Z_{oc}/R_0 with a length of the line 'x'

Short Circuited Line

When the transmission line is shorted from the load end, it is known as short circuited (transmission) line.

The voltage and current equation of a lossless (zero dissipation) transmission line is,

$$V = V_R \cos\beta x + j I_R R_0 \sin\beta x$$

$$I = I_R \cos\beta x + j \frac{V_R}{R_0} \sin\beta x$$

Input Impedance,

$$Z_s = R_0 \left[\frac{Z_R + j R_0 \tan\beta x}{R_0 + j Z_R \tan\beta x} \right]$$

For short circuited line, $Z_R = 0$

replace Z_s by Z_{sc}

Short Circuit Impedance, $Z_{sc} = R_0 \left[\frac{0 + j R_0 \tan\beta x}{R_0 + 0} \right]$

$$= R_0 \left[\frac{j \cancel{R_0} \tan\beta x}{\cancel{R_0}} \right]$$

$$\boxed{Z_{sc} = j R_0 \tan\beta x}$$

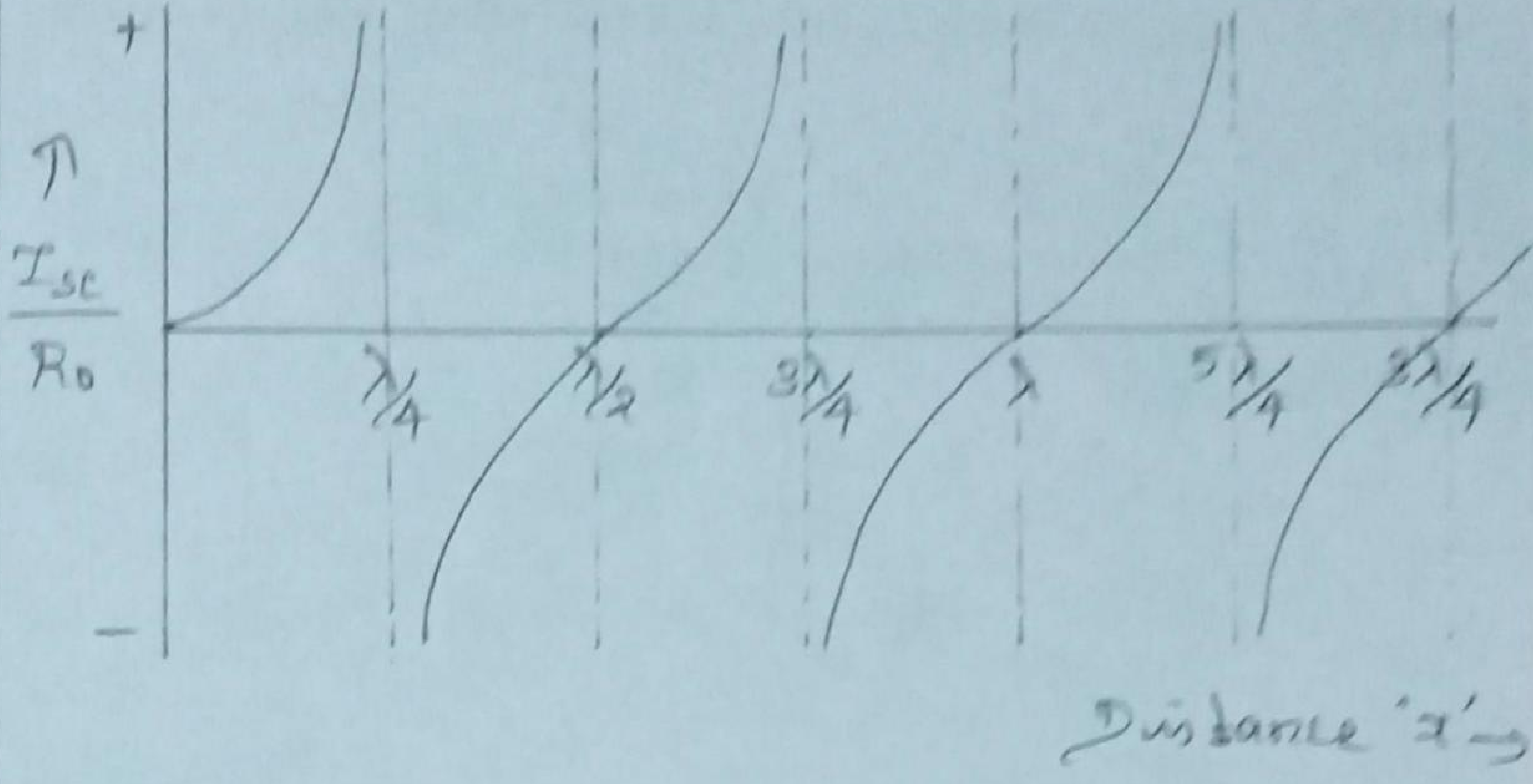

Fig: Variation of $\frac{I_{sc}}{R_0}$ with length of the line 'x'

POWER AND IMPEDANCE MEASUREMENT ON LINES

Power, $P = I^2R$ (or) $P = VI$ (or) $P = \frac{V^2}{R}$

Impedance, $Z = V/I$

Voltage and current equation is,

$$V = \frac{V_R(Z_R + R_0)}{2Z_R}\left[e^{j\beta x} + K e^{-j\beta x}\right]$$

$$I = \frac{I_R(Z_R + R_0)}{2R_0}\left[e^{j\beta x} - K e^{-j\beta x}\right]$$

* maximum of voltage or current occurs along the line at which the incident and reflected waves are inphase

$$V_{max} = \frac{V_R(Z_R + R_0)}{2Z_R}\left[1+|K|\right]$$

$$I_{max} = \frac{I_R(Z_R + R_0)}{2R_0}\left[1+|K|\right]$$

$$\therefore \frac{V_{max}}{I_{max}} = \frac{\frac{V_R(Z_R + \cancel{R_0})}{\cancel{2}Z_R}\left[1+|K|\right]}{\frac{I_R(\cancel{Z_R + R_0})}{\cancel{2}R_0}\left[1+\cancel{|K|}\right]}$$

$$= \frac{\cancel{I_R}\cancel{Z_R}/\cancel{Z_R}}{\cancel{I_R}/R_0}$$

$$\boxed{\therefore \frac{V_{max}}{I_{max}} = R_0 = R_{max}}$$

* minimum of voltage or current occurs along the line at which the incident and reflected waves are out of phase

$$V_{min} = \frac{V_R (Z_R + R_0)}{2 Z_R} \left[1 - |K|\right]$$

$$I_{min} = \frac{I_R (Z_R + R_0)}{2 R_0} \left[1 - |K|\right]$$

$$\frac{V_{min}}{I_{min}} = \frac{\frac{\cancel{I_R} \cancel{Z_R} \cancel{(Z_R + R_0)}}{\cancel{2} \cancel{Z_R}} \cancel{\left[1 - |K|\right]}}{\frac{\cancel{I_R} \cancel{(Z_R + R_0)}}{\cancel{2} R_0} \cancel{\left[1 - |K|\right]}}$$

$$\boxed{\frac{V_{min}}{I_{min}} = R_0 = R_{min}}$$

* Maximum to Minimum Ratio,

$$\frac{V_{max}}{I_{min}} = \frac{\frac{V_R (Z_R + R_0)}{2 Z_R} \left[1 + |K|\right]}{\frac{I_R (Z_R + R_0)}{2 R_0} \left[1 - |K|\right]}$$

$$= \frac{\frac{\cancel{I_R} \cancel{Z_R} \cancel{(Z_R + R_0)}}{\cancel{2} \cancel{Z_R}} \left[1 + |K|\right]}{\frac{\cancel{I_R} \cancel{(Z_R + R_0)}}{\cancel{2} R_0} \left[1 - |K|\right]} = R_0 \frac{\left[1 + |K|\right]}{\left[1 - |K|\right]}$$

$$\boxed{\frac{V_{max}}{I_{min}} = R_0 S}$$

$$\because S = \frac{1 + |K|}{1 - |K|}$$

*Minimum to Maximum Ratio,

$$\frac{V_{min}}{I_{max}} = \frac{\frac{V_R (Z_R \cancel{+ R_0})}{\cancel{2} Z_R}[1-|K|]}{\frac{I_R (Z_R \cancel{+ R_0})}{\cancel{2} R_0}[1+|K|]}$$

$$= \frac{\cancel{I_R} \cancel{Z_R} / \cancel{Z_R}}{\cancel{I_R} / R_0} \frac{[1-|K|]}{[1+|K|]}$$

$$\boxed{\frac{V_{min}}{I_{max}} = \frac{R_0}{S}}$$

$\because S = \frac{1+|K|}{1-|K|}$

Maximum Impedance, $Z_{max} = S R_0$

Minimum Impedance, $Z_{min} = \frac{R_0}{S}$

Power, $$\boxed{P = \frac{V_{max}^2}{R_{max}} \quad \text{(or)} \quad P = \frac{V_{min}^2}{R_{min}}}$$

$$\therefore P^2 = \frac{V_{max}^2 \cdot V_{min}^2}{R_{max} \cdot R_{min}}$$

wkt, $R_{max} = R_{min} = R_0$

$$\Rightarrow P^2 = \frac{V_{max}^2 \cdot V_{min}^2}{R_0 \cdot R_0} = \frac{V_{max}^2 \cdot V_{min}^2}{R_0^2}$$

$$\Rightarrow \boxed{P = \frac{|V_{max}||V_{min}|}{R_0}}$$

(or) $P = I_{max}^2 \cdot R_{max} = I_{max}^2 \cdot R_0 \quad [\because R_{max} = R_0]$

(or) $P = I_{min}^2 \cdot R_{min} = I_{min}^2 \cdot R_0 \quad [\because R_{min} = R_0]$

(or) $P = I_{max} \cdot I_{min} \cdot R_0$

Reflection Losses

When the load impedance is not equal to the characteristic impedance of the transmission line, all the transmitted power will not be delivered to the load and part of the power reflected back to the source. The combination of incident wave and reflected wave gives rise to the standing waves.

Reflection loss occurs on a line which results in part of the transmitted power being reflected back to the source.

* maximum voltage is attained when incident and reflected waves are in phase

$$|V_{max}| = |V_i| + |V_r|$$

where,

V_i - incident voltage wave

V_r - reflected voltage wave

$$= \left|\frac{V_R (Z_R + Z_0)}{2 Z_R}\right| [1 + |K|]$$

$$= \left|\frac{I_R \cancel{Z_R} (Z_R + Z_0)}{2 \cancel{Z_R}}\right| [1 + |K|]$$

$$|V_{max}| = \left|\frac{I_R (Z_R + Z_0)}{2}\right| [1 + |K|]$$

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* minimum voltage is attained when incident and reflected waves are out of phase

$$V_{min} = |V_i| - |V_r|$$

$$= \left|\frac{I_R(Z_R + Z_0)}{2}\right| \left[1 - |K|\right]$$

Standing wave ratio, $S = \frac{|V_{max}|}{|V_{min}|} = \frac{|V_i| + |V_r|}{|V_i| - |V_r|}$

Power delivered to the load, $P = \frac{|V_{max}||V_{min}|}{Z_0}$

$$= \frac{\{|V_i| + |V_r|\}\{|V_i| - |V_r|\}}{Z_0}$$

$$P = \frac{|V_i|^2 - |V_r|^2}{Z_0}$$

let,

P_i - transmitted power in the incident wave

P_r - reflected power in the reflected wave

Power delivered to the load, $P = P_i - P_r$

The ratio of power delivered to the load to the power transmitted by the incident wave is,

$$\frac{P}{P_i} = \frac{P_i - P_r}{P_i} = 1 - \frac{P_r}{P_i}$$

Measurement of VSWR and Wavelength

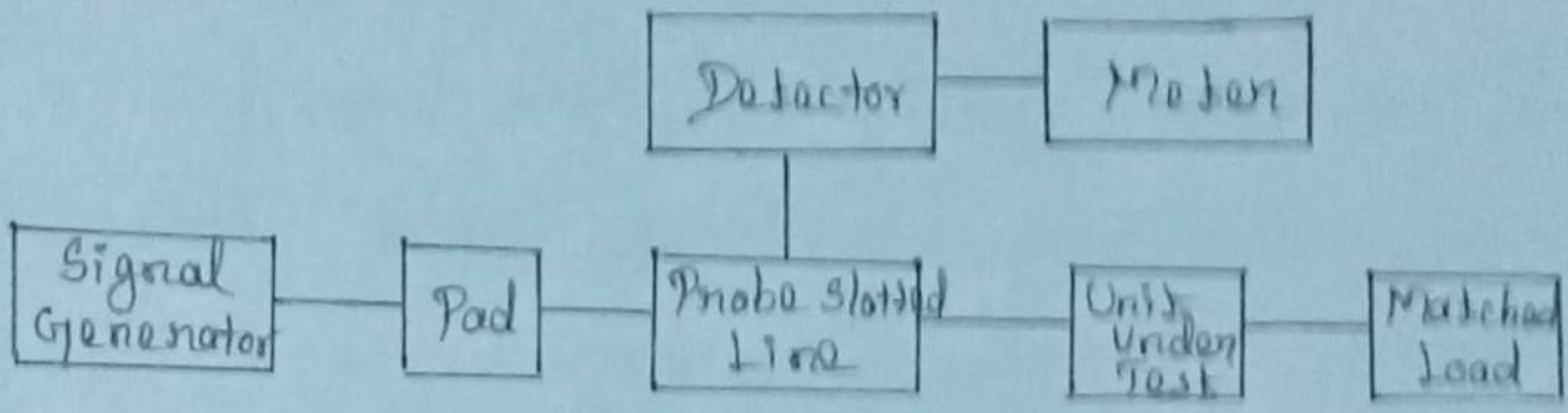

Fig: VSWR Measurement

The signal source must generate a signal matching the impedance, else reflection will occur which initiates shift proportion in peaks and nodes. To match with the generator, a pad is attached between the signal source and equipment.

The pad attenuates the input signals as it is resistive and reflection absorbent. If the level of the signal is low to admit attenuation, the resistive pad will not be effective as minimum voltage readings will be hired by instrument noise.

The standing wave ratio measured at the input will be the component under test. If the component absorbs all the power, the matched load will terminate it. The output generated will be a mismatch due to reflection detected at slotted probe line. If the tested (device) component is an absorbing device, then matched load is not a requirement at the output.

Each and every frequency produces a different VSWR, maximum and minimum position for a frequency modulated or spurious signals which leads to improper reading. Square wave modulations are used by source for exact measurements.

Standing wave indicator is used as output meter for detection. It is an audio amplified calibrated for VSWR measurement. The meter reads the full scale when the voltage is maximum adjusted by the pad. Assuming a square law detector and placing the probe to a minimum point. VSWR meter reads the values directly.

UNIT - III

IMPEDANCE MATCHING IN HIGH FREQUENCY LINES

Impedance matching: Quarter wave transformer - Impedance matching by stubs - single stub matching and Double stub matching - Smith Chart - Solutions of problems using Smith Chart - Single and Double stub matching using Smith chart.

IMPEDANCE MATCHING

Impedance matching is a process of matching the load impedance with (source) characteristic impedance.

Impedance matching is designing source and load impedances to minimize signal reflection or maximize power transfer.

DC Circuits

* Source and load should be equal

AC Circuits

* Source should either equal the load or complex conjugate of the load

Impedance matching is achieved by making the load impedance equal to the source impedance. If the source impedance, load impedance and transmission line characteristic impedance are purely resistive, than reflectionless matching is the same as maximum power transfer matching.

Impedance Matching Circuits

* L - network
* π - network
* Split capacitor network
* Transmatch circuit

The transmission lines of different wavelength can be used as impedance matching and impedance transformation.

* Coaxial cable
* Balun Transformer
* Matching Stubs
* Quarter Wavelength Transformer
* Series Matching Section

Need for Impedance Match

* Preventing signal reflection
* Maximum Power Transfer
* Make the driver/receiver impedance resistive

Quarterwave Transformer

Also named as quarterwave line or $\lambda/4$ line or impedance inverter. If the length of the transmission line is $1/4^{th}$ of wavelength is termed as $\lambda/4$ line or quarterwave line. Quarterwave line act as impedance inverter/ impedance transformer.

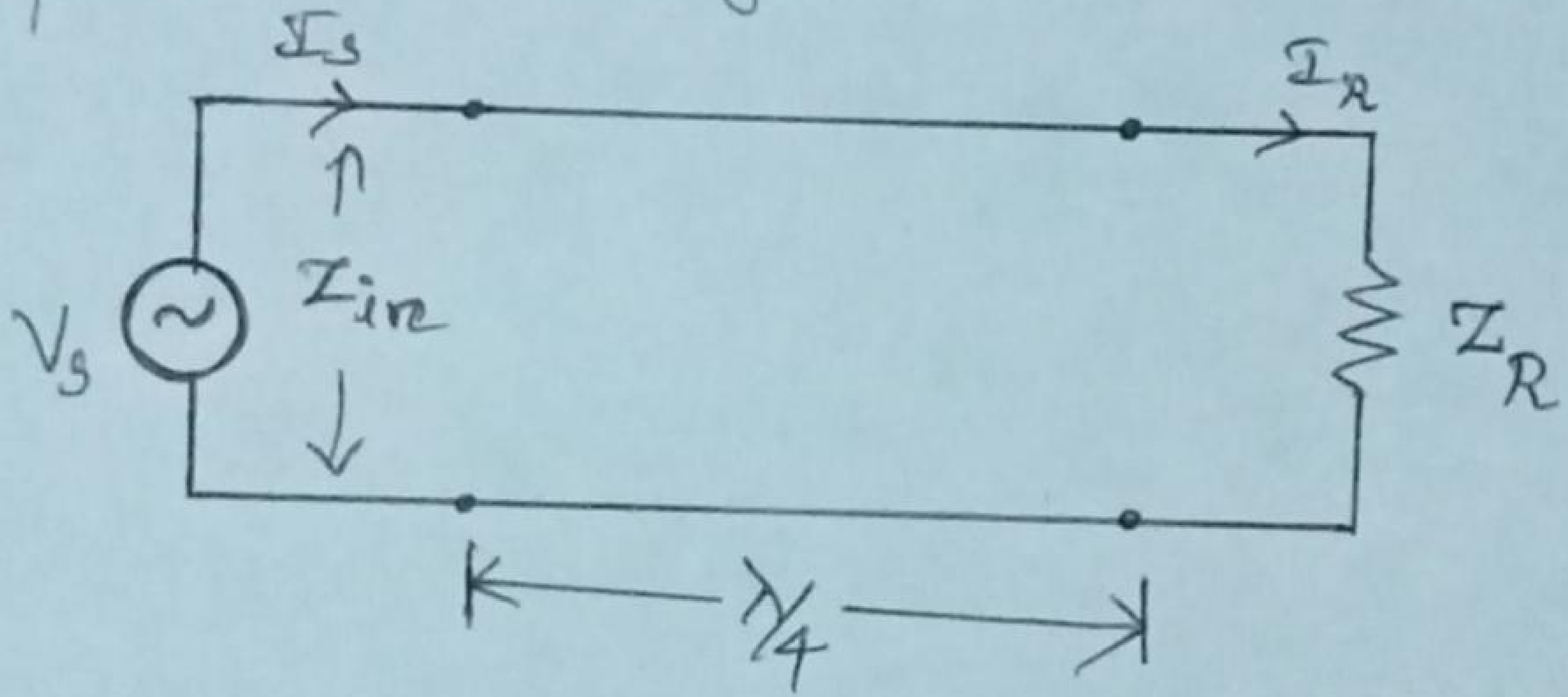

Fig: Quarterwave Line

The general expression for the input impedance of a dissipationless transmission line is,

$$Z_s = R_0 \left[\frac{Z_R + jR_0 \tan\beta x}{R_0 + jZ_R \tan\beta x} \right]$$

$$= R_0 \cdot \frac{\cancel{\tan\beta x} \left[\frac{Z_R}{\tan\beta x} + jR_0 \right]}{\cancel{\tan\beta x} \left[\frac{R_0}{\tan\beta x} + jZ_R \right]}$$

$$Z_s = R_0 \frac{\left[\frac{Z_R}{\tan\beta x} + jR_0\right]}{\left[\frac{R_0}{\tan\beta x} + jZ_R\right]}$$

For quarterwave line,

length, $x = \lambda/4$

Phase Constant, $\beta = \frac{2\pi}{\lambda}$

now, $$Z_s = R_0 \frac{\left[\frac{Z_R}{\tan\frac{\cancel{2}\pi}{\cancel{\lambda}}\cdot\frac{\cancel{\lambda}}{\cancel{4}_2}} + jR_0\right]}{\left[\frac{R_0}{\tan\frac{\cancel{2}\pi}{\cancel{\lambda}}\cdot\frac{\cancel{\lambda}}{\cancel{4}_2}} + jZ_R\right]}$$

$$= R_0 \frac{\left[\frac{Z_R}{\tan\pi/2} + jR_0\right]}{\left[\frac{R_0}{\tan\pi/2} + jZ_R\right]}$$

$$= R_0 \left[\frac{\frac{Z_R}{\infty} + jR_0}{\frac{R_0}{\infty} + jZ_R}\right]$$ $\left[\because \tan\pi/2 = \tan 90^\circ = \infty\right]$ $\left[\because 1/\infty = 0\right]$

$$= R_0 \left[\frac{0 + jR_0}{0 + jZ_R}\right]$$

$$= R_0 \cdot \frac{\cancel{j}R_0}{\cancel{j}Z_R}$$

$$\boxed{Z_s = \frac{R_0^2}{Z_R}}$$

$$\Rightarrow R_0^2 = Z_s Z_R$$

$$\boxed{R_0 = \sqrt{Z_s Z_R}}$$

where,

R_0 - Characteristic impedance

Z_s - Source (input) impedance

Z_R - Load impedance

For matching impedances Z_s and Z_R, the transmission line with characteristic R_0 should be selected such that condition $R_0 = \sqrt{Z_s Z_R}$ gets satisfied.

Quarterwave line can transform a low impedance into a high impedance and high impedance into a low impedance, thus it can be considered as an impedance inverter.

Impedance Matching by stubs

stub - finite length transmission line

Impedance matching is achieved by the use of open or short circuited line of suitable length, called stub at a designated distance from the load is called stub matching.

Types

1. Single stub matching
2. Double stub matching

Single stub Matching

Impedance matching is achieved with the help of single open circuited or short circuited stub is named as single stub matching.

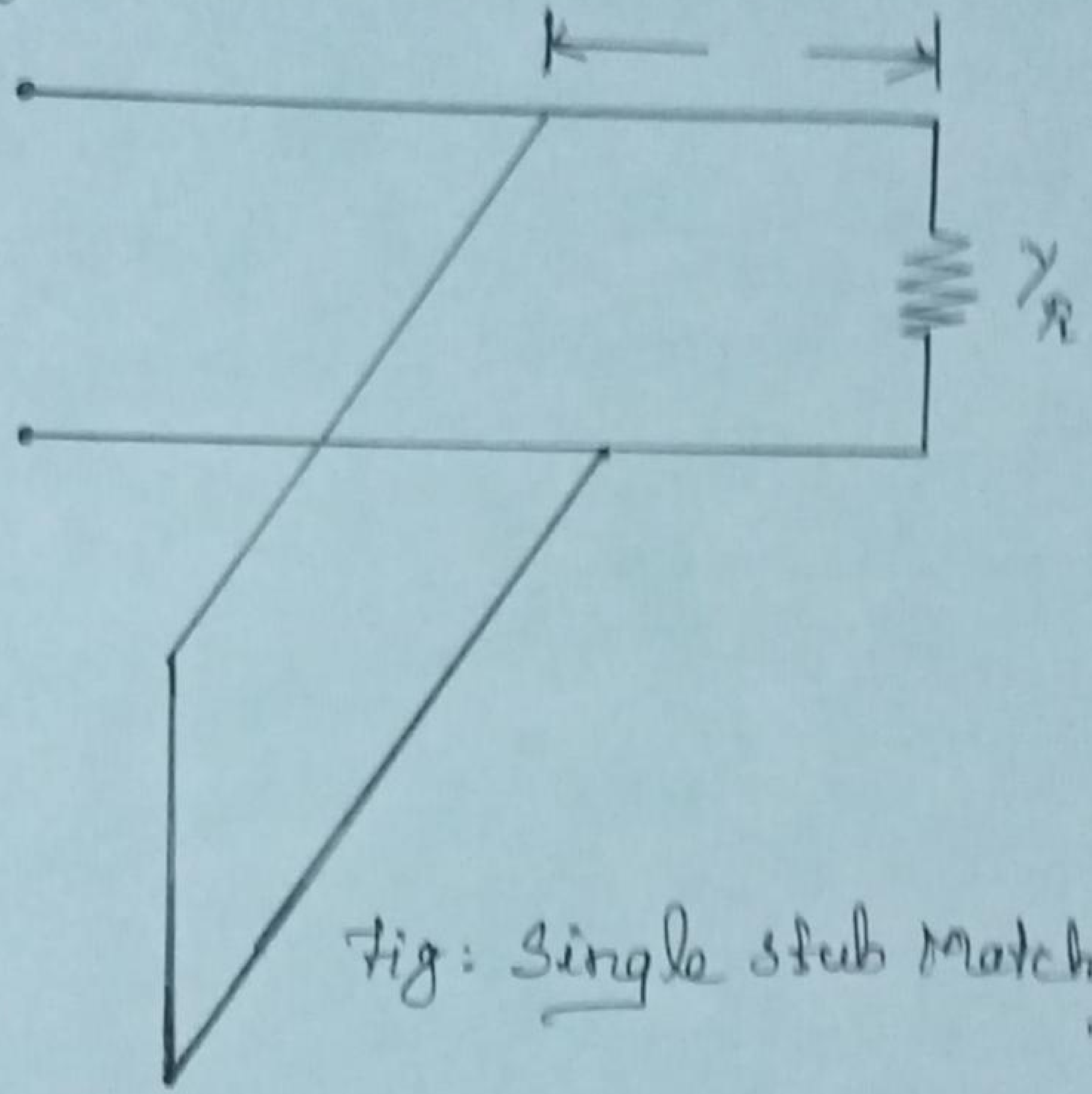

Fig: Single stub Matching

A transmission line having characteristic admittance Y_0 terminated with load admittance Y_R. Since Y_R is different from Y_0 [ie, $Y_R = Y_0$], standing waves are set up in between source and load.

The input impedance at any point of a transmission line is,

$$Z_s = Z_0 \left\{ \frac{Z_R + Z_0 \tanh \gamma l}{Z_0 + Z_R \tanh \gamma l} \right\}$$

The input admittance is,

$$Y_s = Y_0 \left\{ \frac{Y_R + Y_0 \tanh \gamma l}{Y_0 + Y_R \tanh \gamma l} \right\}$$

For propagation $\gamma = j\beta$, $\alpha = 0$,

$$Y_s = Y_0 \left\{ \frac{Y_R + jY_0 \tan \beta l}{Y_0 + jY_R \tan \beta l} \right\}$$

$$\Rightarrow \frac{Y_s}{Y_0} = \frac{\cancel{Y_0}\left\{ \frac{Y_R}{Y_0} + j\tan\beta l \right\}}{\cancel{Y_0}\left\{ 1 + j\frac{Y_R}{Y_0}\tan\beta l \right\}}$$

$$\frac{Y_s}{Y_0} = \frac{Y_r + j\tan\beta l}{1 + jY_r \tan\beta l}$$

$$\Rightarrow Y_{in} = \frac{Y_r + j\tan\beta l}{1 + jY_r\tan\beta l}$$

where,

$\frac{Y_s}{Y_0} = Y_{in} \rightarrow$ normalized input admittance

$\frac{Y_R}{Y_0} = Y_r \rightarrow$ normalized load admittance

To separate the real and imaginary parts,

$$Y_{in} = \frac{Y_r + j\tan\beta l}{1 + jY_r\tan\beta l} \times \frac{1 - jY_r\tan\beta l}{1 - jY_r\tan\beta l}$$

$$= \frac{y_r - j y_r^2 \tan\beta l + j \tan\beta l - j^2 y_r \tan^2\beta l}{1 - j^2 y_r^2 \tan^2\beta l}$$

$$= \frac{y_r + y_r \tan^2\beta l + j \tan\beta l - j y_r^2 \tan\beta l}{1 + y_r^2 \tan^2\beta l}$$

$$y_{in} = \frac{y_r(1+\tan^2\beta l) + j \tan\beta l (1 - y_r^2)}{1 + y_r^2 \tan^2\beta l}$$

$$\Rightarrow y_{in} = \frac{y_r(1+\tan^2\beta l)}{1+y_r^2 \tan^2\beta l} + j \frac{\tan\beta l (1-y_r^2)}{1+y_r^2\tan^2\beta l}$$

For perfect matching, $Y_{in} = Y_0$

$$\Rightarrow \frac{Y_{in}}{Y_0} = 1$$

$$\therefore y_{in} = 1$$

So the stub has to be located at a point where the real part of y_{in} is equal to unity.

<u>Location of the Stub (l_s)</u>

* real part of $y_{in} = 1$

i.e.,

$$\frac{y_R(1+\tan^2\beta l_s)}{1+y_r^2\tan^2\beta l_s} = 1$$

$$y_R(1+\tan^2\beta l_s) = 1 + y_r^2 \tan^2\beta l_s$$

$$y_r + y_r \tan^2\beta l_s - y_r^2 \tan^2\beta l_s = 1$$

$$y_r \tan^2\beta l_s (1 - y_r) = 1 - y_r$$

$$y_r \tan^2\beta l_s = \frac{\cancel{(1-y_r)}}{\cancel{(1-y_r)}}$$

$$\tan^2 \beta l_s = \frac{1}{Y_r}$$

$$\tan \beta l_s = \frac{1}{\sqrt{Y_r}}$$

$$\beta l_s = \tan^{-1}\left[\frac{1}{\sqrt{Y_r}}\right]$$

$$\frac{2\pi}{\lambda} l_s = \tan^{-1}\left[\frac{1}{\sqrt{Y_r}}\right] \qquad \left[\because \beta = \frac{2\pi}{\lambda}\right]$$

$$l_s = \frac{\lambda}{2\pi} \tan^{-1}\left(\frac{1}{\sqrt{Y_r}}\right)$$

we know that, $Y_r = Y_R / Y_0$

$$\therefore \; l_s = \frac{\lambda}{2\pi} \tan^{-1} \frac{1}{\sqrt{Y_R/Y_0}}$$

location of the stub, $\boxed{l_s = \frac{\lambda}{2\pi} \tan^{-1}\sqrt{\frac{Y_0}{Y_R}}}$

(or) location of the stub, $\boxed{l_s = \frac{\lambda}{2\pi} \tan^{-1}\sqrt{\frac{Z_R}{Z_0}}}$

Lengthe of the stub (l_t)

To find the length of the stub the imaginary part (susceptance) is equated to $\cot \beta l_t$.

$$\frac{\tan \beta l_t (1 - Y_r^2)}{1 + Y_r^2 \tan^2 \beta l_t} = \cot \beta l_t$$

substitute $\tan \beta l = \frac{1}{\sqrt{Y_r}} = \sqrt{\frac{Y_0}{Y_R}}$

$$\frac{\sqrt{\frac{Y_0}{Y_R}}\,(1-y_r^2)}{1+y_r^2\cdot\left(\sqrt{\frac{Y_0}{Y_R}}\right)^2} = \cot\beta l_s$$

$$\Rightarrow \cot\beta l_s = \frac{\sqrt{\frac{Y_0}{Y_R}}\left\{1-\left(\frac{Y_R}{Y_0}\right)^2\right\}}{1+y_r^2\cdot\frac{Y_0}{Y_R}}$$

$$= \frac{\sqrt{\frac{Y_0}{Y_R}}\left\{1-\frac{Y_R^2}{Y_0^2}\right\}}{1+\cancel{y_r^2}\cdot\frac{1}{\cancel{y_r}}} \qquad \left[\because y_r = \frac{Y_R}{Y_0}\right]$$

$$= \frac{\sqrt{\frac{Y_0}{Y_R}}\left\{\frac{Y_0^2-Y_R^2}{Y_0^2}\right\}}{1+y_r}$$

$$= \frac{\sqrt{\frac{Y_0}{Y_R}}\;\frac{(Y_0+Y_R)(Y_0-Y_R)}{Y_0^2}}{1+\frac{Y_R}{Y_0}}$$

$$= \frac{\sqrt{\frac{Y_0}{Y_R}}\;\cancel{\left(\frac{Y_0+Y_R}{Y_0}\right)}\left(\frac{Y_0-Y_R}{Y_0}\right)}{\cancel{\left(\frac{Y_0+Y_R}{Y_0}\right)}}$$

$$= \sqrt{\frac{\cancel{Y_0}}{Y_R}}\cdot\frac{(Y_0-Y_R)}{\cancel{\sqrt{Y_0}}\,\sqrt{Y_0}}$$

$$\cot\beta l_s = \frac{Y_0-Y_R}{\sqrt{Y_0 Y_R}}$$

length of the stub, $\boxed{l_s = \frac{\lambda}{2\pi}\cot^{-1}\left(\frac{Y_0-Y_R}{\sqrt{Y_0 Y_R}}\right)}$

$$\cot \beta l_t = \frac{Y_0 - Y_R}{\sqrt{Y_0 Y_R}}$$

$$\frac{1}{\tan \beta l_t} = \frac{Y_0 - Y_R}{\sqrt{Y_0 Y_R}}$$

$$\tan \beta l_t = \frac{\sqrt{Y_0 Y_R}}{Y_0 - Y_R}$$

$$\beta l_t = \tan^{-1}\left(\frac{\sqrt{Y_0 Y_R}}{Y_0 - Y_R}\right)$$

$$\frac{2\pi}{\lambda} l_t = \tan^{-1}\left(\frac{\sqrt{Y_0 Y_R}}{Y_0 - Y_R}\right)$$

$$\boxed{l_t = \frac{\lambda}{2\pi} \tan^{-1}\left(\frac{\sqrt{Y_0 Y_R}}{Y_0 - Y_R}\right)}$$

(or)

$$l_t = \frac{\lambda}{2\pi} \tan^{-1}\left(\frac{\sqrt{\frac{1}{Z_0} \cdot \frac{1}{Z_R}}}{\frac{1}{Z_0} - \frac{1}{Z_R}}\right)$$

$$= \frac{\lambda}{2\pi} \tan^{-1} \frac{\sqrt{1/Z_0 Z_R}}{\frac{Z_R - Z_0}{Z_0 Z_R}}$$

$$= \frac{\lambda}{2\pi} \tan^{-1}\left(\frac{Z_0 Z_R}{(Z_R - Z_0)\sqrt{Z_0 Z_R}}\right)$$

$$= \frac{\lambda}{2\pi} \tan^{-1} \frac{\sqrt{Z_0 Z_R}\,\cancel{\sqrt{Z_0 Z_R}}}{(Z_R - Z_0)\,\cancel{\sqrt{Z_0 Z_R}}}$$

length of the stub

$$\boxed{l_t = \frac{\lambda}{2\pi} \tan^{-1}\left(\frac{\sqrt{Z_0 Z_R}}{Z_R - Z_0}\right)}$$

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Double Stub Matching

Impedance matching is achieved by using two open circuited or short circuited stubs is termed as double stub matching.

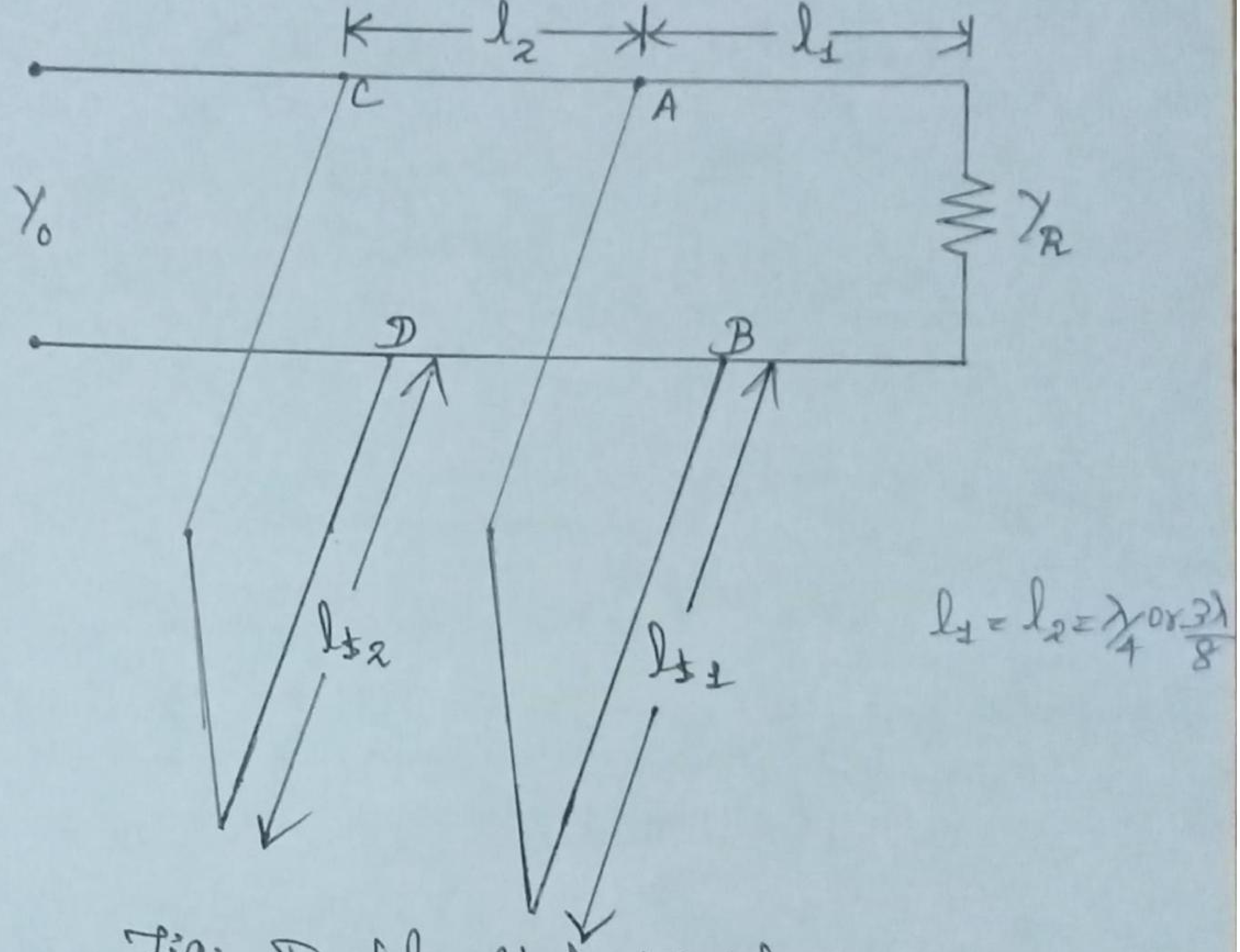

Fig: Double Stub Matching

To avoid the disadvantages of single stub matching, double stub matching is introduced. Double stub matching is one in which two short circuited stubs spacing $\lambda/4$ or $3\lambda/8$ whose lengths are adjustable independently are fixed.

Let the first stub of length l_{s1} be located at AB on the transmission line at a distance l_1 from the load.

The normalised input impedance,

$$y_{AB} = \frac{Y_{AB}}{Y_0} = \frac{y_r + j\tan\beta l_1}{1 + j y_r \tan\beta l_1}$$

$$\Rightarrow y_{AB} = \frac{y_r + j\tan\beta l_1}{1 + j y_r \tan\beta l_1} \times \frac{1 - j y_r \tan\beta l_1}{1 - j y_r \tan\beta l_1}$$

$$= \frac{y_r + j\tan\beta l_1 - j y_r^2 \tan\beta l_1 + y_r^2 \tan^2\beta l_1}{1 + y_r^2 \tan^2\beta l_1}$$

$$= \frac{y_r + y_r \tan\beta l_1 + j\tan\beta l_1 - j y_r^2 \tan^2\beta l_1}{1 + y_r^2 \tan^2\beta l_1}$$

$$y_{AB} = \frac{y_r(1 + \tan^2\beta l_1) + j\tan\beta l_1 (1 - y_r^2)}{1 + y_r^2 \tan^2\beta l_1}$$

we know that,

Admittance, Y = Conductance, g + Susceptance, jB

$$\therefore y_{AB} = g_{AB} + j b_{AB}$$

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$$\therefore Y_{AB} = g_{AB} + j b_{AB} = \frac{y_1 (1+\tan^2 \beta l_1)}{1+y_1^2 \tan^2 \beta l_1} + \frac{j(1-y_1^2)\tan\beta l_1}{1+y_1^2 \tan^2 \beta l_1}$$

$$\Rightarrow g_{AB} = \frac{y_1 (1+\tan^2 \beta l_1)}{1+y_1^2 \tan^2 \beta l_1}$$

$$b_{AB} = \frac{(1-y_1^2)\tan\beta l_1}{1+y_1^2 \tan^2 \beta l_1}$$

When the second stub is introduced at 'CD' at the distance l_2 from 'AB', the susceptance value is changed but conductance value is unchanged.

i.e., $g_{AB} = g_{CD}$ & $b_{AB} \neq b_{CD}$

In practice the location of stub AB is always less than $\lambda/2$. Sometimes the first stub is located at the load itself.

In general,

→ distance $l_1 = 0.1\lambda$ to 0.15λ

→ distance between two stub is either $\frac{3\lambda}{8}$ or $\frac{\lambda}{4}$.

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SMITH CHART

Smith chart is a polar diagram containing constant resistance circles and constant reactance circles used for solving transmission line and waveguide problems.

Smith chart is a tool for representing complex impedances in polar coordinates. It is used for designing feedlines, filters, antennas etc.

Smith chart is based on two sets of orthogonal circles. The tangents drawn at the points of intersection of two circles ie. resistance and reactance circles.

To display the impedance of all possible passive networks the graph must extend in all possible directions $(R, jx, -jx)$. The smith chart is committed to a bilinear transformation by plotting the complex reflection co-efficient.

Reflection co-efficient, $K = \frac{Z_R - Z_0}{Z_R + Z_0}$

By normalize,

$$= \frac{\cancel{Z_0}\left(\frac{Z_R}{Z_0} - 1\right)}{\cancel{Z_0}\left(\frac{Z_R}{Z_0} + 1\right)}$$

$$\Rightarrow \boxed{K = \frac{Z_r - 1}{Z_r + 1}}$$

where,

$Z_r = \frac{Z_R}{Z_0} \rightarrow$ Normalized Load Impedance

$$\Rightarrow K(Z_r + 1) = Z_r - 1$$

$$K Z_r + K = Z_r - 1$$

$$1 + K = Z_r - K Z_r$$

$$1 + K = Z_r (1 - K)$$

$$\Rightarrow Z_r (1 - K) = 1 + K$$

$$\boxed{Z_r = \frac{1+K}{1-K}}$$

here, Z - complex quantity $\rightarrow Z = R + jX$

$\therefore K$ - also complex quantity $\rightarrow K = K_R + jK_X$

now $Z_r = \frac{1+K}{1-K}$ becomes,

$$R + jX = \frac{1 + K_R + jK_X}{1 - K_R - jK_X}$$

To separate real and imaginary terms,

$$R + jX = \frac{(1 + K_R) + jK_X}{(1 - K_R) - jK_X} \times \frac{(1 - K_R) + jK_X}{(1 - K_R) + jK_X}$$

$$= \frac{1 - \cancel{K_R} + jK_X + \cancel{K_R} - K_R^2 + j\cancel{K_X} K_R + jK_X - j\cancel{K_X} K_R + j^2 K_X^2}{(1 - K_R)^2 - j^2 K_X^2}$$

$$= \frac{1 + 2jK_X - K_R^2 - K_X^2}{(1 - K_R)^2 + K_X^2}$$

$$R+jX = \frac{(1-K_R^2-K_x^2)+j2K_x}{(1-K_R)^2+K_x^2}$$

$$R+jX = \frac{1-K_R^2-K_x^2}{(1-K_R)^2+K_x^2} + j\frac{2K_x}{(1-K_R)^2+K_x^2}$$

Equating the real part on both sides,

$$R = \frac{1-K_R^2-K_x^2}{(1-K_R)^2+K_x^2}$$

Equating the imaginary parts on both sides,

$$X = \frac{2K_x}{(1-K_R)^2+K_x^2}$$

R-Circles

Consider the real part,

$$R = \frac{1-K_R^2-K_x^2}{(1-K_R)^2+K_x^2}$$

$$R\left[(1-K_R)^2+K_x^2\right] = 1-K_R^2-K_x^2$$

$$R(1-2K_R+K_R^2)+RK_x^2 = 1-K_R^2-K_x^2$$

$$R-2RK_R+RK_R^2+RK_x^2+K_R^2+K_x^2 = 1$$

Rearrange,

$$\underbrace{K_R^2+RK_R^2}+\underbrace{K_x^2+RK_x^2}-2RK_R = 1-R$$

$$K_R^2(1+R)+K_x^2(1+R)-2RK_R = 1-R$$

$$(1+R)\left\{K_R^2+K_x^2-\frac{2RK_R}{1+K_R}\right\} = 1-R$$

$$\Rightarrow \quad K_R^2 + K_X^2 - \frac{2RK_R}{1+R} = \frac{1-R}{1+R}$$

$$K_R^2 - \frac{2RK_R}{1+R} + K_X^2 = \frac{1-R}{1+R}$$

$(a-b)^2 = a^2 - 2ab + b^2$, $a = K_R$, $b = \frac{R}{1+R} \Rightarrow \left(K_R - \frac{R}{1+R}\right)^2 = K_R^2 - \frac{2RK_R}{1+R} + \left(\frac{R}{1+R}\right)^2$

$\Rightarrow$ Above equation $\left(\frac{R}{1+R}\right)^2$ is missing,

$\Rightarrow \therefore$ add and subtract $\left(\frac{R}{1+R}\right)^2$ in above equation,

$$\underbrace{K_R - \frac{2RK_R}{1+R} + \left(\frac{R}{1+R}\right)^2} - \left(\frac{R}{1+R}\right)^2 + K_X^2 = \frac{1-R}{1+R}$$

$$\left(K_R - \frac{R}{1+R}\right)^2 + K_X^2 = \frac{1-R}{1+R} + \frac{R^2}{(1+R)^2}$$

$$\left(K_R - \frac{R}{1+R}\right)^2 + K_X^2 = \frac{(1-R)(1+R) + R^2}{(1+R)^2}$$

$$\left(K_R - \frac{R}{1+R}\right)^2 + K_X^2 = \frac{1 - \cancel{R^2} + \cancel{R^2}}{(1+R)^2}$$

$$\boxed{\left(K_R - \frac{R}{1+R}\right)^2 + K_X^2 = \frac{1}{(1+R)^2}}$$

This is the equation of family of 'R' circles.

centre $= \left(\frac{R}{1+R}, 0\right)$, Radius $= \frac{1}{1+R}$

X-circles

Consider the imaginary part,

$$X = \frac{2K_X}{(1-K_R)^2 + K_X^2}$$

$$x\left[(1-K_R)^2+K_X^2\right] = 2K_X$$

$$x - 2xK_R + xK_R^2 + xK_X^2 - 2K_X = 0$$

Dividing by 'x',

$$1 - 2K_R + K_R^2 + K_X^2 - \frac{2K_X}{x} = 0$$

Rearrange,

$$\underbrace{K_R^2 - 2K_R + 1} + \underbrace{K_X^2 - 2\frac{K_X}{x}}_{1/x^2 \text{ is missing}} = 0$$

Add and subtract $1/x^2$ we get,

$$(K_R - 1)^2 + \underbrace{K_X^2 - \frac{2K_X}{x} + \frac{1}{x^2}} - \frac{1}{x^2} = 0$$

$$\boxed{(K_R - 1)^2 + \left(K_X - \frac{1}{x}\right)^2 = \frac{1}{x^2}}$$

This is the equation of family of 'x' circles.

centre $= 1, 1/x$, Radius $= 1/x$

(i) when 'x' is positive

$\Rightarrow$ the circle lies above the (horizontal) real axis

(ii) when 'x' is negative

$\Rightarrow$ the circle lies below the (horizontal) real axis

(iii) when 'x' is zero

$\Rightarrow$ the circle becomes a straight line along the real axis

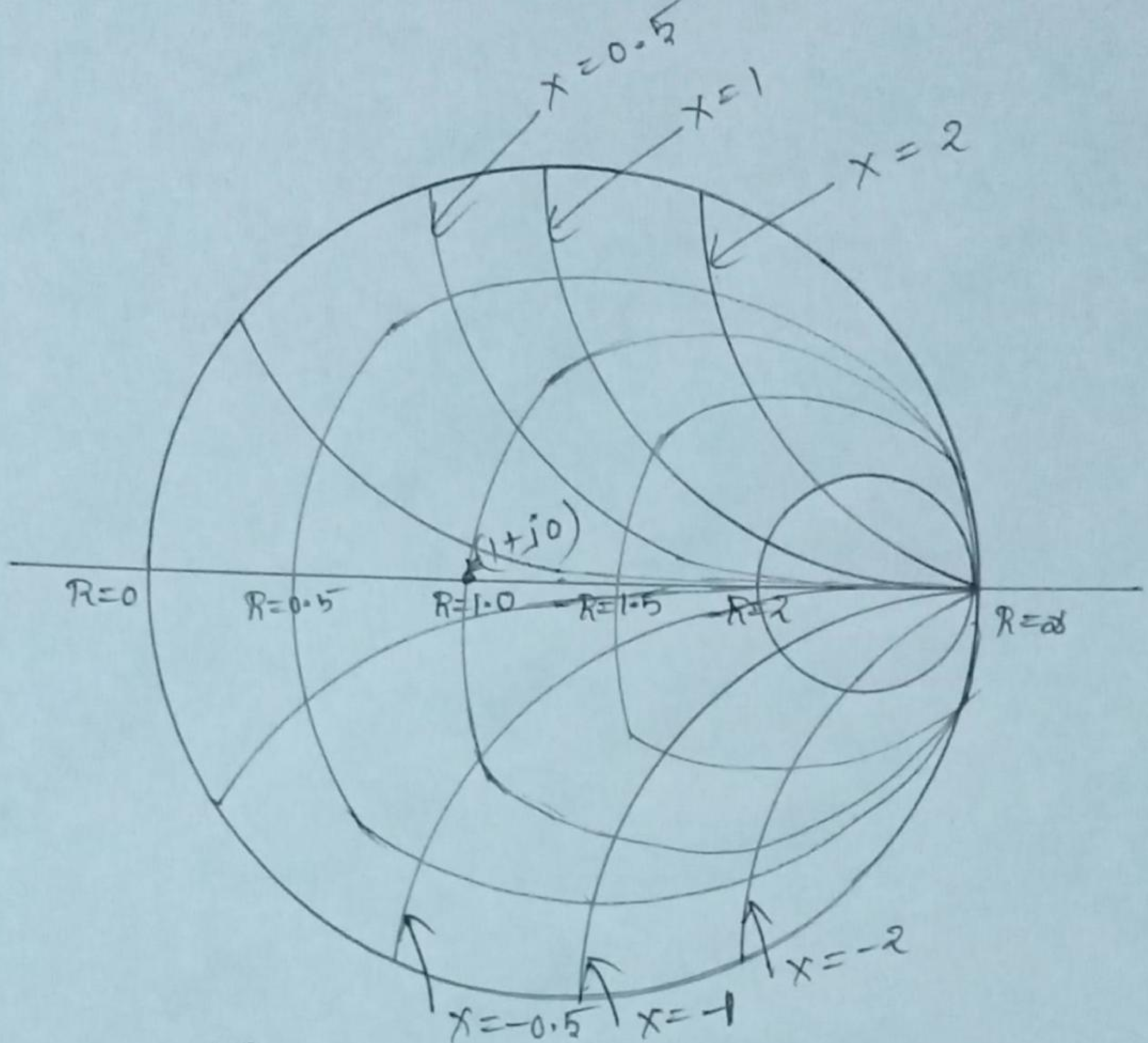

Fig: Smith Circle Diagram

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Solutions of Problems Using Smith chart

Single and Double Stub Matching Using Smith chart

Problems

17.a. Antenna with impedance $40+j70\ \Omega$ is to be matched to a 100Ω lossless line with a shorted stub. Determine the following using Smith chart. Nov/Dec - 2017

a) The required stub admittance

b) The distance between the stub and the antenna

c) The stub length

d) The standing wave ratio on each of the system.

<u>Solution</u>

Given, Characteristic impedance, $Z_0 = 160\ \Omega$

Load impedance, $Z_L = 40+j70\ \Omega$

Normalized load impedance, $z_L = \dfrac{40+j30}{100}$

$= 0.4+j0.7\ \Omega$

The normalized load impedance is plotted on Smith chart as 'A' and the impedance circle is also drawn.

From the chart SWR is obtained as,

$SWR = 7.2$

The normalized load admittance is diametrically opposite to the normalized load impedance at 'B'.

ie, $Y_L = 1.85 - j1.4$

The admittance point is rotated clockwise to a point 'c' on the impedance circle where it intersects the $R=1$ circle. At this point admittance, $\underline{y} = 1+j1.2$

The distance between the point 'C' and point 'B' is the distance of stub from the load (d).

$$d = (0.5\lambda - 0.296) + 0.167\lambda = 0.204\lambda + 0.167\lambda$$
$$= 0.371\lambda$$

The stub must have zero resistance and an susceptance that has an exactly opposite value of D is, $\underline{y}_{stub} = 0 - j1.2$

The length of the stub between E and origin (X=0) 'l',

$$l = 0.36\lambda - 0.25\lambda$$
$$= 0.11\lambda$$

Answer

1. Stub Admittance, $Y = \frac{1.85 - j1.4}{100}$

$$= 0.0185 - j0.014 \ \mho$$

2. Distance between the stub and antenna $\}$ $d = 0.371\lambda$

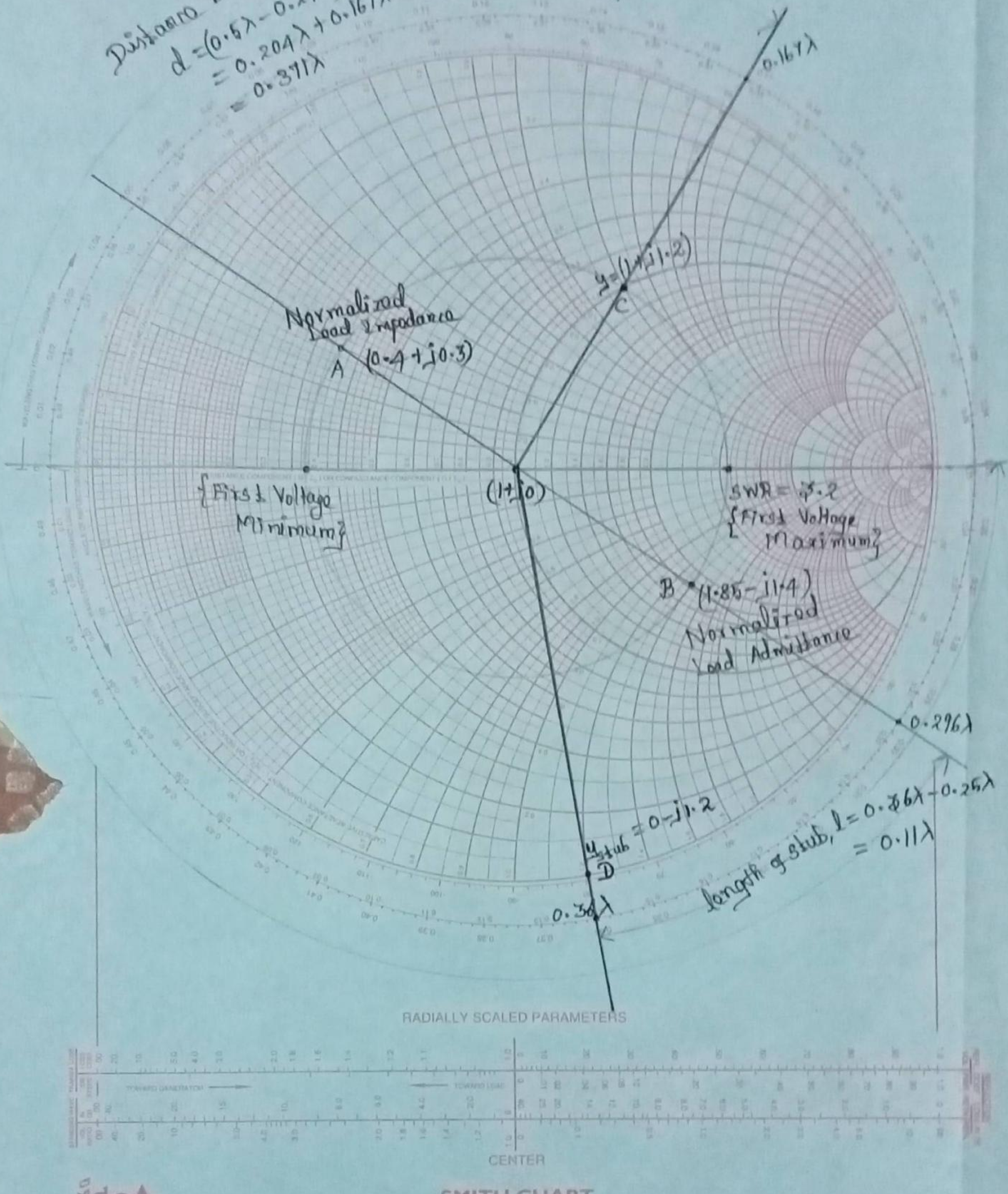
3. Stub length, $l = 0.11\lambda$

4. Standing Wave Ratio, SWR = 3.2

17.a.

Distance between stub and antenna

$$d = (0.5\lambda - 0.296\lambda) + 0.167\lambda$$
$$= 0.204\lambda + 0.167\lambda$$
$$= 0.371\lambda$$



length of stub, $l = 0.36\lambda - 0.25\lambda$
$= 0.11\lambda$

14-b. Design a double stub shunt tuner to match a load impedance $Z_L = 60-j80\,\Omega$ to a $50\,\Omega$ line. The stubs are to be short-circuited stubs and are spaced $\lambda/8$ apart. Find the lengths of the two stubs using Smith Chart.
NOV/DEC-2017

Solution

Given, $Z_0 = 50\,\Omega$, $Z_L = 60-j80\,\Omega$

Normalized load impedance, $z_L = \dfrac{Z_L}{Z_0}$

$$= \frac{60-j80}{50}$$

$$= 1.2 - j1.6\,\Omega$$

Normalized load admittance, $y_L = \dfrac{Y_L}{Y_0} = \dfrac{1/Z_L}{1/Z_0}$

$$= \frac{Z_0}{Z_L}$$

$$= \frac{50}{60-j80} \times \frac{60+j80}{60+j80}$$

$$= \frac{50(60+j80)}{60^2 - j^2 80^2}$$

$$= \frac{3000+j4000}{3600+6400}$$

$$= \frac{3000+j4000}{10{,}000}$$

$$y_L = 0.3 + j0.4\ \mho$$

* The load admittance is marked on the smith chart.
* Circle A for $Y/Y_0 = 1$ is drawn.
* Distance between the two stub is $\lambda/8$, $\lambda/8 = 90°$.
* Circle A is rotated by $90°$, new circle 'B' is drawn.

* Drag the point $(0.8+j0.4)$ to circle 'B' and reach at $y_{s_1} = 0.8 + j1.7$

The change of reactance is,

$$y_{s_1} - y_L = (0.8+j1.7) - (0.8+j0.4)$$
$$= j1.3$$

* The reactance of $j1.3$ is marked on the smith chart (B_{s1}) at 0.145λ.
* The length of the stub 1 is measured from short circuit point to B_{s1} point.

$$l_{t_1} = 0.25\lambda + 0.145\lambda$$
$$= 0.395\lambda$$

* The point y_{s1} is moved along the 'S' circle to reach the unit circle at a point $y_{s_2} = 1+j3.6$.
* The reactance at y_{s2} is $j3.6$. This reactance is cancelled by placing negative susceptance of $-j3.6$ in the opposite side. i.e., $B_{s2} = -j3.6$ at 0.296λ.

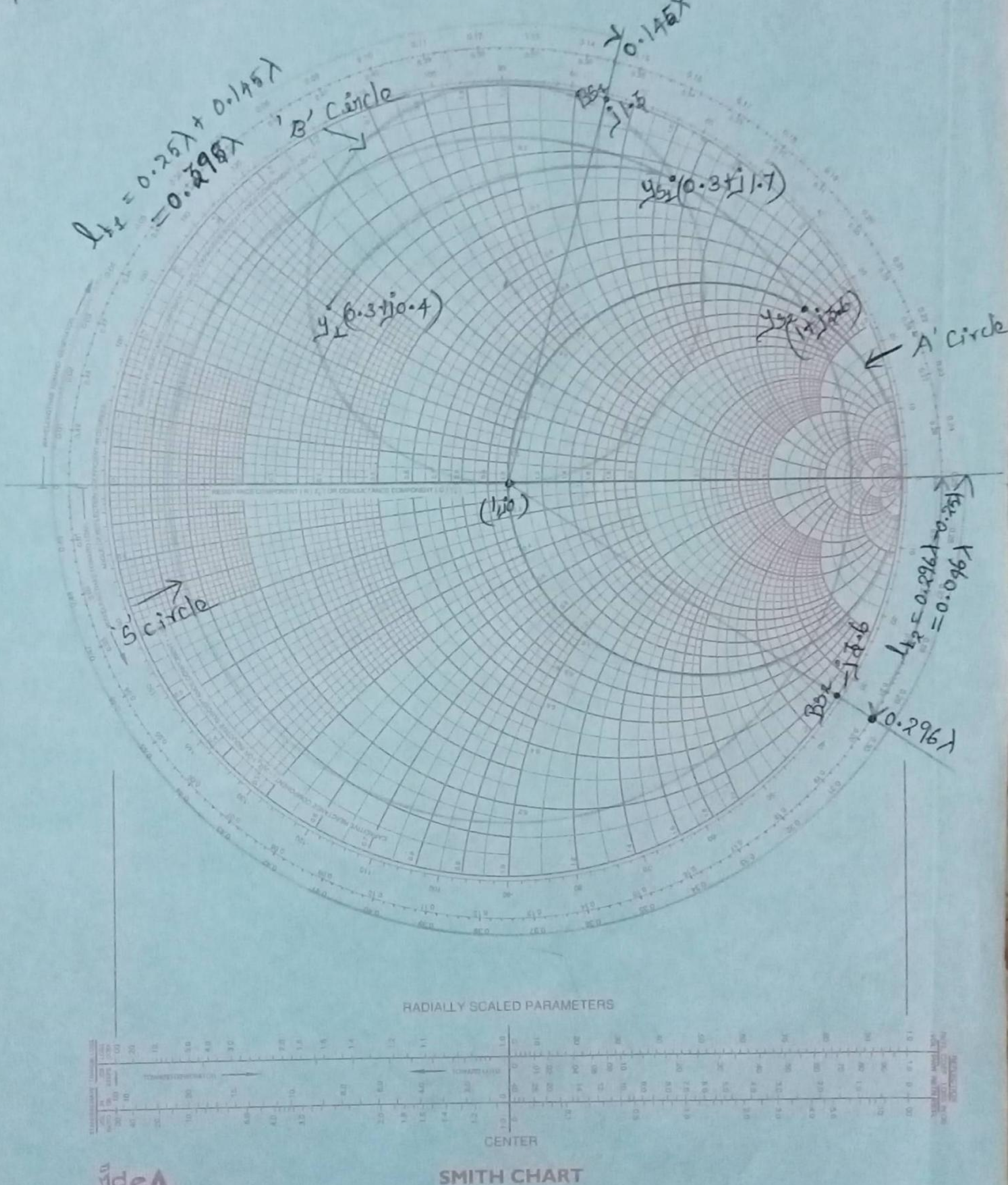
The length of stub 2, $l_{t_2} = 0.296\lambda - 0.25\lambda$
$$= 0.046\lambda$$

Answer

Stub 1 length, $l_{t_1} = 0.395\lambda$

Stub 2 length, $l_{t_2} = 0.046\lambda$

17.b.



$l_{s1} = 0.25\lambda + 0.145\lambda = 0.395\lambda$
'B' Circle
λ 0.145λ
Bs1 $j1.5$
y_{s1} (0.3+j1.7)
y_L (0.3+j0.4)
y_{s2} (1+j2.6)
'A' Circle
(1, j0)
'S' circle
$l_{s2} = 0.296\lambda - 0.25\lambda = 0.046\lambda$
Bs2 $-j2.6$
0.296λ
RADIALLY SCALED PARAMETERS
CENTER
SMITH CHART

12.a) Design a single stub matching network (use smith chart) for a transmission line functiong at 1500 MHz terminated with a load impedance $Z_L = (300 + j250)\,\Omega$ and with a characteristic impedance $Z_0 = 100$ ohms. Use short circuited shunt stubs. Determine the VSWR before and after connecting the stub.

May/June - 2013

<u>Solution</u>

Given, $Z_0 = 100\,\Omega$, $f = 1500\,MHz$

$$Z_L = 300 + j250$$

$$\text{Normalized load impedance} = \frac{300 + j250}{100}$$

$$= (3 + j2.5)\,\Omega$$

$$\lambda = c/f = \frac{3\times10^8}{\frac{1500\times10^6}{3}}$$

$$= 0.6\,m$$

* Taking the centre point (1,0) and the radius as the distance between the centre and the normalized load impedance point (3.00 + j 2.5) the impedance circle is drawn.

* The point diametrically opposite to the normalized load impedance point on the impedance circle is the normalized load admittance.

* The admittance point is rotated clockwise on the impedance circle where it intersects R = 1 circle. At this point admittance $y = 1 + j1.9$

* The distance between this point and the load admittance is the location of the stub.

$$l_s = 0.472\lambda - 0.184\lambda$$
$$= 0.288\lambda$$

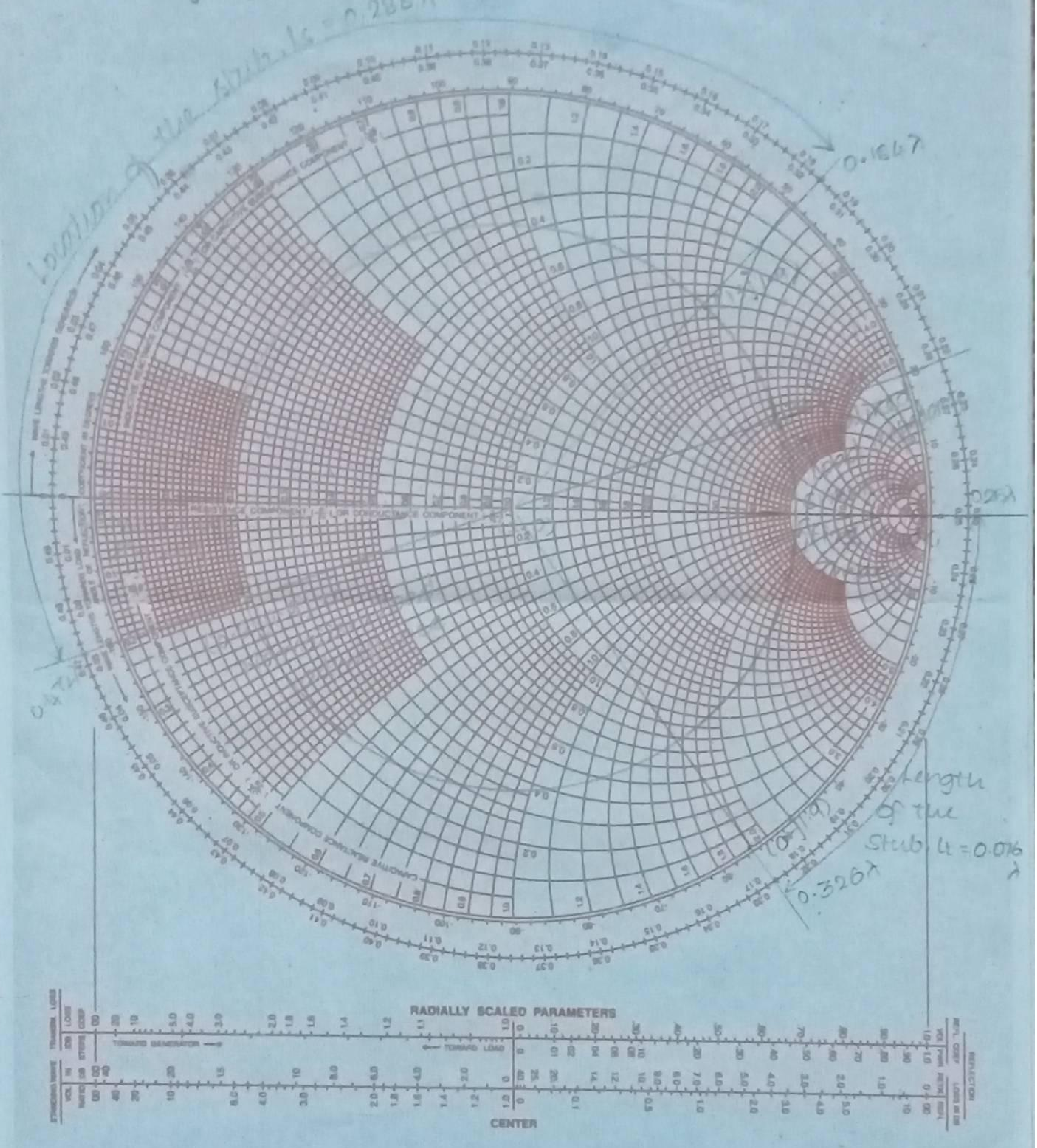
* The stub must have zero resistance and an susceptance that has an exactly opposite value of 'y'.

$$y_{new} = (0 - j1.9)\ \mho$$

$$l_t = 0.326\lambda - 0.25\lambda$$
$$= 0.076\lambda$$

$Z_0 = 100\,\Omega$

$Z_L = 300 + j250\,\Omega$



SMITH CHART

What is Quarter wave line?

A line of $\lambda/4$ length is called as quarterwave line.

Quarter wave line can be used as,

* an impedance inverter
* couple a transmission line to a load
* serve as an insulator to support an open wire line

Input impedance, $Z_s = R_0^2 / Z_R$

ii) A 75Ω lossless transmission line is to be matched with a (100-j80)Ω load using single stub. Calculate the stub length and its distance from the load corresponding to the frequency of 30 MHz using Smith Chart. NOV/DEC-2015

Solution

Given, $Z_0 = 75\,\Omega$, $Z_R = (100-j80)\,\Omega$

$f = 30\,MHz$

$\lambda = c/f = \frac{3\times10^8}{30\times10^6}$

$= 10m$

Normalized load impedance, $Z_1 = \frac{Z_R}{Z_0}$

$= \frac{100-j80}{75}$

$= (1.33 - j1.067)\,\Omega$

* Taking the centre point (1,0) and the radius as the distance between the centre and the normalized load impedance point (1.33-j1.067), the impedance circle is drawn.

* The point diametrically opposite to the normalized load impedance point on the impedance circle is the normalized load admittance.

* The admittance point is rotated clockwise on the impedance circle where it intersects R=1 circle. At this point admittance $y = 1 + j0.95$

* The distance between this point and the load admittance is the location of the stub.

Distance between $\left(\substack{0.46 \\ + \\ j0.38}\right)$ to $(1+j0.95)$

i.e., $l_s = 0.16\lambda - 0.068\lambda = 0.092\lambda$

* The stub must have zero resistance and an susceptance that has an exactly opposite value of 'y'.

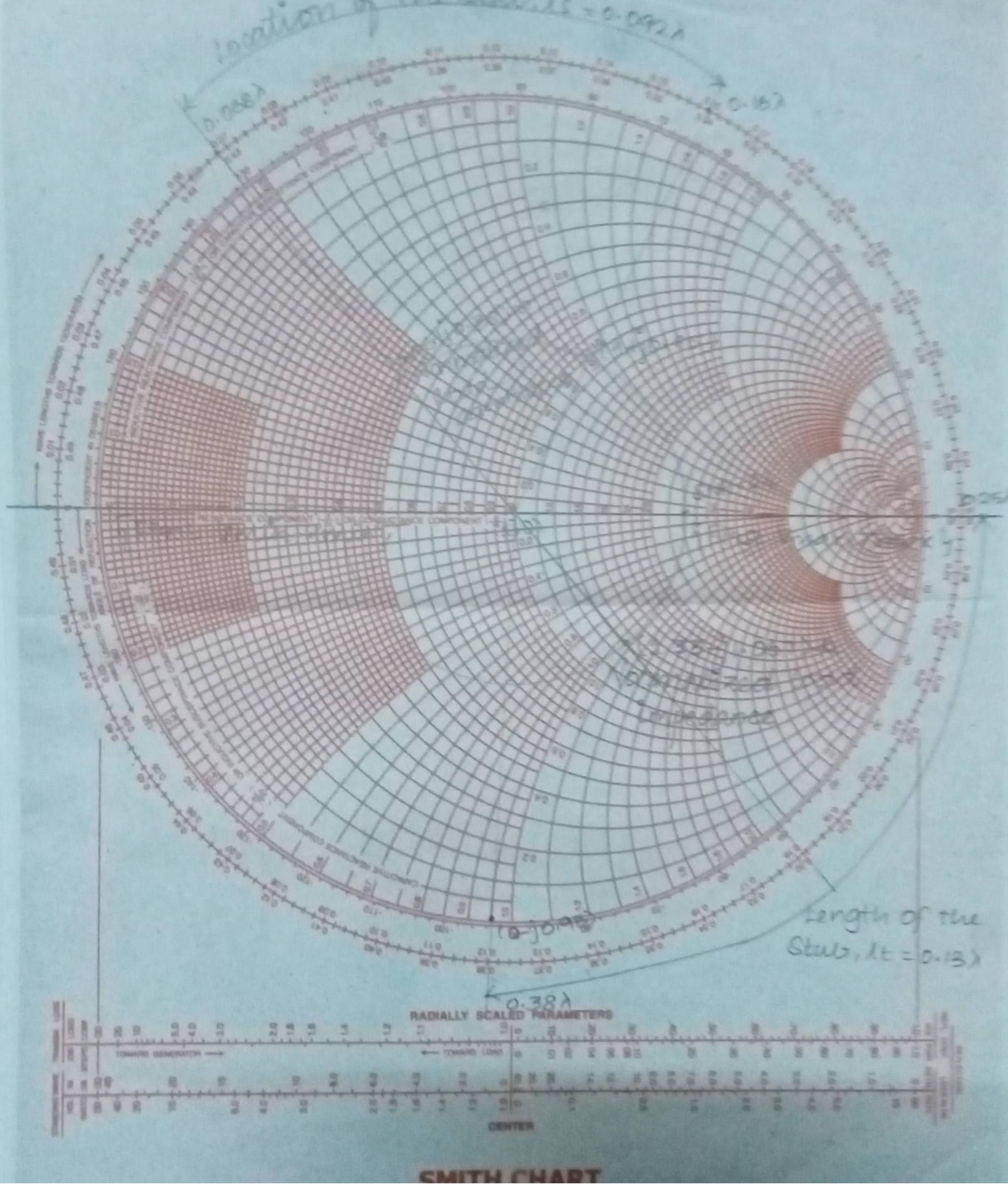
$$y_{new} = (0 - j0.95)\,\mho$$

Distance between V_{max} to $(0 - j0.95)\,\mho$

i.e., $l_t = 0.78\lambda - 0.25\lambda$
$= 0.13\lambda$

$Z_0 = 75\,\Omega$

$Z_L = 100 - j80\,\Omega$



SMITH CHART

ii) A 300Ω transmission line is connected to a load impedance of (450 − j600) Ω at 10 MHz. Find the position and length of a short circuited stub required to match the line using smith chart.

Nov/Dec-2015

Solution

Given, $Z_0 = 300\,\Omega$, $f = 10\,MHz$

$Z_R = 450 - j600$

Normalized load impedance, $z_L = \dfrac{Z_R}{Z_0}$

$= \dfrac{450 - j600}{300}$

$= (1.5 - j2)\,\Omega$

* Taking the centre point (1,0) and the radius as the distance between the centre and normalized load impedance point (1.5 − j2), the impedance circle is drawn.

* The point diametrically opposite to the normalized load impedance point on the impedance circle is the normalized load admittance.

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* The admittance point is rotated clockwise on the impedance circle where it intersects R = 1 circle. At this point admittance $y = 1 + j1.7$

* The distance between this point and the load admittance is the location of the stub.

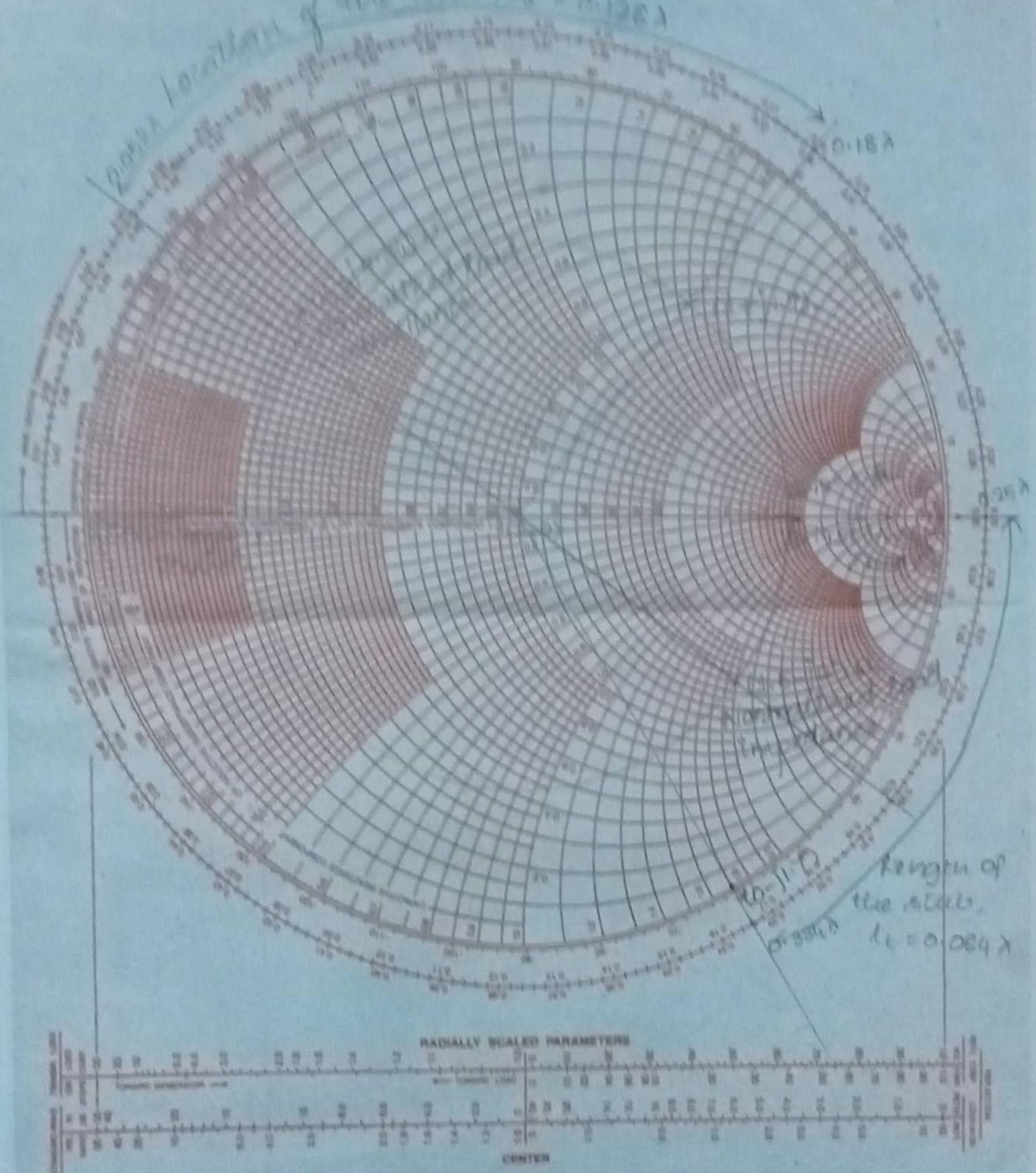
ie., $l_s = 0.18\lambda - 0.052\lambda$
$= 0.128\lambda$

* The stub must have zero resistance and an susceptance that has an exactly opposite value of 'y'.

$y_{new} = 0 - j1.7$

$\therefore l_1 = 0.334\lambda - 0.25\lambda$
$= 0.084\lambda$

$Z_L = (400 - j600)\,\Omega$
$Z_0 = 300\,\Omega$



SMITH CHART

UNIT IV

WAVEGUIDES

General wave behavior along uniform guiding structures - Transverse Electromagnetic waves, Transverse Magnetic Waves, Transverse Electric Waves - TM and TE waves between parallel plates. Field equations in rectangular waveguides. TM and TE waves in rectangular waveguides, Bessel Functions. TM and TE waves in circular waveguides.

GENERAL WAVE BEHAVIOR ALONG UNIFORM GUIDING STRUCTURE

Waveguides are used to transfer electro-magnetic waves from one place to another. Some common guiding structures are, co-axial cable, two wire lines, hollow conducting waveguides and optical fibers.

Parallel Plane

* wave propagates along z-direction

General field equations of an electromagnetic wave propagates between parallel planes are,

$$H_x = \frac{-\gamma}{h^2} \frac{\partial H_z}{\partial x}$$

$$H_y = \frac{-j\omega\varepsilon}{h^2} \frac{\partial E_z}{\partial x}$$

$$E_x = \frac{-\gamma}{h^2} \frac{\partial E_z}{\partial x}$$

$$E_y = \frac{j\omega\mu}{h^2} \frac{\partial H_z}{\partial x}$$

Rectangular Wave guide

General field equations of an electromagnetic wave propagates within the rectangular waveguide are,

$$E_x = \frac{-\gamma}{h^2} \frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2} \frac{\partial H_z}{\partial y}$$

$$E_y = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}$$

$$H_x = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial x} + \frac{j\omega\epsilon}{h^2}\frac{\partial E_z}{\partial y}$$

$$H_y = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\epsilon}{h^2}\frac{\partial E_z}{\partial x}$$

Circular Waveguide

General field equations of an electromagnetic wave propagates within the circular waveguide are,

$$E_\rho = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial \rho} - \frac{j\omega\mu}{\rho h^2}\frac{\partial H_z}{\partial \phi}$$

$$E_\phi = \frac{-\gamma}{\rho h^2}\frac{\partial E_z}{\partial \phi} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial \rho}$$

$$H_\rho = \frac{j\omega\epsilon}{h^2}\frac{\partial E_z}{\partial \phi} - \frac{\gamma}{h^2}\frac{\partial H_z}{\partial \rho}$$

$$H_\phi = \frac{-j\omega\epsilon}{h^2}\frac{\partial E_z}{\partial \rho} - \frac{\gamma}{\rho h^2}\frac{\partial H_z}{\partial \phi}$$

TRANSVERSE ELECTROMAGNETIC WAVES

Transverse electromagnetic waves are waves in which both electric and magnetic fields are transverse entirely but has no component of E_z and H_z. It is also referred to as principal waves.

Transverse electromagnetic is a mode of propagation where the electric and magnetic field lines are all restricted to directions normal [transverse] to the direction of propagation.

TEM, also referred to as transmission line mode, is the principal mode of wave propagation and exists only in transmission lines made of two conductors. This mode becomes dominant in wave propagation where the cross-sectional area of the transmission line is small compared to the signal wavelength.

Consider the field strength for TM waves,

$$H_y = C_4 \cos\left(\frac{m\pi}{a}x\right) e^{-j\beta z}$$

$$E_x = \frac{\beta}{\omega \epsilon} C_4 \cos\left(\frac{m\pi}{a}x\right) e^{-j\beta z}$$

$$E_z = \frac{jm\pi}{\omega \epsilon a} C_4 \sin\left(\frac{m\pi}{a}x\right) e^{-j\beta z}$$

For TEM waves, $E_z = 0$ and the minimum value of $m = 0$.

$$\therefore H_y = C_4 \cos\left(0 \times \frac{\pi}{a}x\right) e^{-j\beta z}$$

$$E_x = \frac{\beta}{\omega \epsilon} C_4 \cos\left(0 \times \frac{\pi}{a}x\right) e^{-j\beta z}$$

$$E_z = 0$$

$$\Rightarrow H_y = C_4 (\cos 0) e^{-j\beta z} = C_4 e^{-j\beta z} \quad [\because \cos 0 = 1]$$

$$\Rightarrow E_x = \frac{\beta}{\omega \epsilon} C_4 (\cos 0) e^{-j\beta z} = \frac{\beta}{\omega \epsilon} C_4 e^{-j\beta z}$$

$$E_z = 0$$

These fields are not only entirely transverse, but they are constant in amplitude between parallel planes.

TRANSVERSE MAGNETIC WAVES IN PARALLEL PLANES

Transverse magnetic waves are waves in which the magnetic field strength is entirely transverse. It has an electric field strength in the direction of propagation and no component of magnetic field in the same direction.

i.e., $H_z = 0$, $E_z \neq 0$

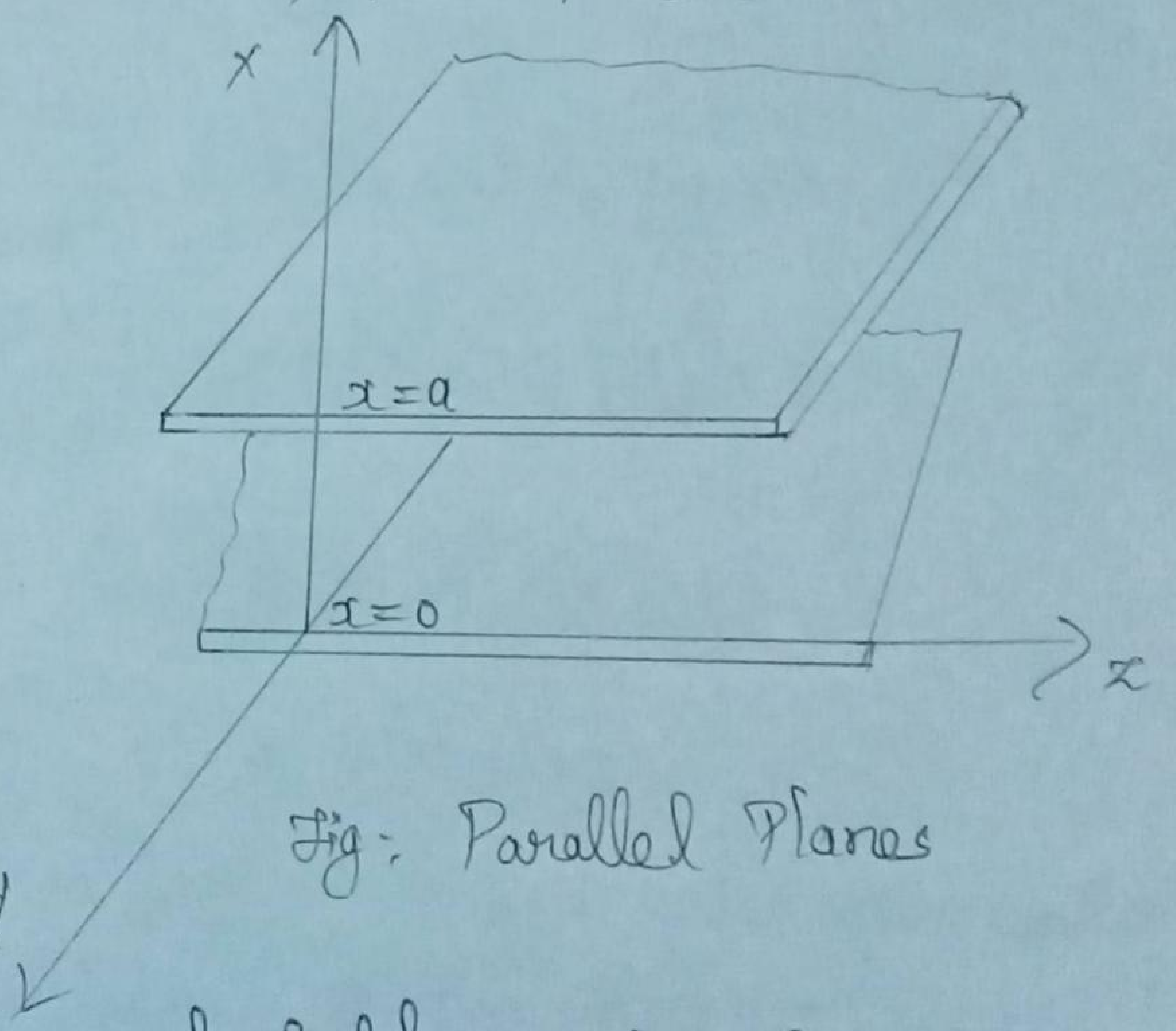


Fig: Parallel Planes

The general field equations of an EM wave between parallel planes are,

$$H_x = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial x} \longrightarrow (1)$$

$$H_y = \frac{-j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x} \longrightarrow (2)$$

$$E_x = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x} \rightarrow (3)$$

$$E_y = \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x} \rightarrow (4)$$

Substitute $H_z = 0$,

$H_x = 0$, $E_y = 0$ but $E_x \neq 0$, $H_y \neq 0$

The wave equation for the component H_y,

$$\frac{\partial^2 H_y}{\partial x^2} + \gamma^2 H_y = -\omega^2\mu\varepsilon H_y \rightarrow (5)$$

$$\frac{\partial^2 H_y}{\partial x^2} + \gamma^2 H_y + \omega^2\mu\varepsilon H_y = 0$$

$$\frac{\partial^2 H_y}{\partial x^2} + (\gamma^2 + \omega^2\mu\varepsilon) H_y = 0$$

$$\frac{\partial^2 H_y}{\partial x^2} + h^2 H_y = 0 \rightarrow (6)$$

where, $h^2 = \gamma^2 + \omega^2\mu\varepsilon$

This is [equ (6)] second order differential equation. The solution of this equation is,

$$H_y = C_1 \sin hx + C_2 \cos hx \rightarrow (7)$$

C_1, C_2 - arbitrary constants

$\hookrightarrow$ determined from boundary conditions

$\Rightarrow$ H_y in terms of time and direction,

$$H_y = (C_1 \sin hx + C_2 \cos hx)e^{-\gamma z} \rightarrow (8)$$

[$\because$ H_y - vector quantity]

* The tangential component of 'H' is not zero at the surface of a conductor
* Therefore the boundary conditions cannot be applied directly to H_y to determine the arbitrary constants 'c_1' and 'c_2'
* $\therefore$ E_z can be obtained in terms of H_z

we know that,

$$\frac{\partial H_y}{\partial x} = j\omega\epsilon E_z$$

[refer general wave behaviour along uniform guiding structure]

$$\Rightarrow E_z = \frac{1}{j\omega\epsilon}\frac{\partial H_y}{\partial x}$$

$$= \frac{1}{j\omega\epsilon}\frac{\partial}{\partial x}\left[c_1 \sinh hx + c_2 \cosh hx\right]e^{-\gamma z}$$

$$= \frac{1}{j\omega\epsilon}\left[c_1 \cosh hx.h + c_2 (-\sinh hx)h\right]e^{-\gamma z}$$

$$\Rightarrow E_z = \frac{h}{j\omega\epsilon}\left[c_1 \cos hx - c_2 \sin hx\right]e^{-\gamma z} \rightarrow (9)$$

Boundary conditions are, $x=0$, $x=a$

i) $E_z = 0$ when $x=0$

$$\therefore E_z = \frac{h}{j\omega\epsilon}\left[c_1 \cos 0 - c_2 \sin 0\right]e^{-\gamma z} = 0$$

This is possible [i.e., $E_z = 0$] only when $c_1 = 0$

now the solution (equ. 9) becomes,

$$E_z = \frac{-h}{j\omega\epsilon_1} C_2 \sinh x . e^{-\gamma z} \longrightarrow (10)$$

ii) $E_z = 0$ when $x = a$,

$$(10) \Rightarrow \quad E_z = \frac{-h}{j\omega\epsilon_1} C_2 \sin ha . e^{-\gamma z} = 0$$

This is possible only when $h = \frac{m\pi}{a}$, $m = 1, 2, 3, \ldots$
(ie, $E_z = 0$)

now the solution becomes [equ. 10],

$$\boxed{E_z = \frac{-m\pi}{j\omega\epsilon_1 a} C_2 \sin\left(\frac{m\pi}{a}\right)x . e^{-\gamma z}} \longrightarrow (11)$$

Substitute $C_1 = 0$, $h = \frac{m\pi}{a}$ in equ. (8),

$$\boxed{H_y = C_2 \cos\left(\frac{m\pi}{a}\right)x . e^{-\gamma z}} \longrightarrow (12)$$

wkt,

$$\gamma H_y = j\omega\epsilon_1 E_x$$

$$\Rightarrow E_x = \frac{\gamma}{j\omega\epsilon_1} H_y$$

$$\Rightarrow \boxed{E_x = \frac{\gamma}{j\omega\epsilon_1} C_2 \cos\left(\frac{m\pi}{a}\right)x . e^{-\gamma z}} \longrightarrow (13)$$

γ - propagation constant

where, $\gamma = \alpha + j\beta$

* If the wave propagates without attenuation, $\alpha = 0$, only 'β' exists.

∴ The field strengths for TM waves between parallel planes are,

$$\boxed{\begin{aligned} H_y &= C_2 \cos\left(\frac{m\pi}{a}\right)x \cdot e^{-j\beta z} \\ E_x &= \frac{\beta}{\omega\varepsilon} C_2 \cos\left(\frac{m\pi}{a}\right)x \cdot e^{-j\beta z} \\ E_z &= \frac{-m\pi}{j\omega\varepsilon a} C_2 \sin\left(\frac{m\pi}{a}\right)x \cdot e^{-j\beta z} \end{aligned}}$$

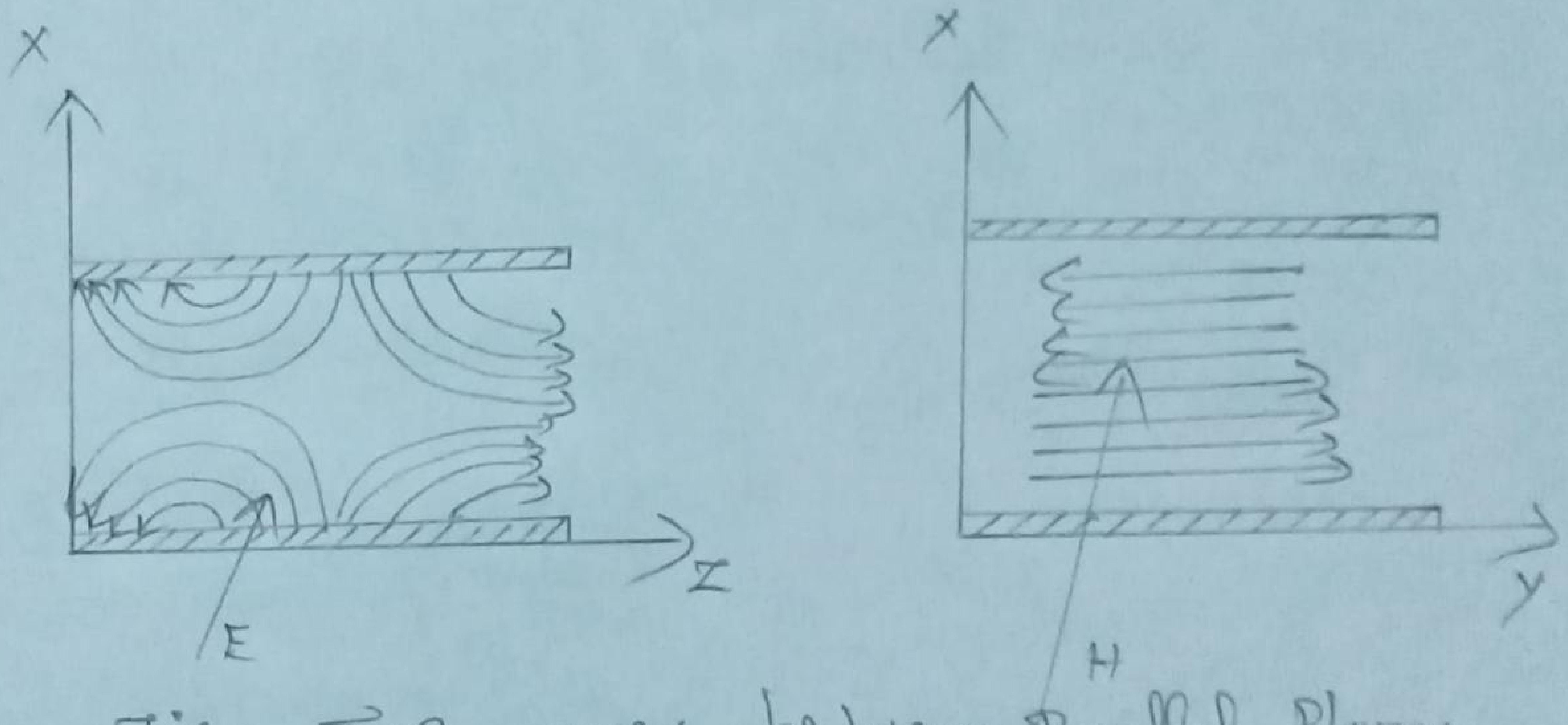


Fig: T.M waves between Parallel Planes

TRANSVERSE ELECTRIC WAVES IN PARALLEL PLANES

Transverse electric waves are waves in which the electric field strength is entirely transverse. It has a magnetic field strength in the direction of propagation and no component of electric field in the same direction.

i.e., $E_z = 0,\ H_z \neq 0$

The electric and magnetic field strengths of an EM wave between parallel planes are,

$$H_x = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial x} \longrightarrow (1)$$

$$H_y = \frac{-j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x} \longrightarrow (2)$$

$$E_x = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x} \longrightarrow (3)$$

$$E_y = \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x} \longrightarrow (4)$$

For TE waves, $E_z = 0$,

now,
$$H_x = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial x}$$

$$H_y = 0$$

$$E_x = 0$$

$$E_y = \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}$$

The field equations of TE waves in parallel planes are,

$$H_x = \frac{-\gamma}{h^2} \frac{\partial H_z}{\partial x} \longrightarrow (5)$$

$$E_y = \frac{j\omega\mu}{h^2} \frac{\partial H_z}{\partial x} \longrightarrow (6)$$

and $H_z \neq 0$

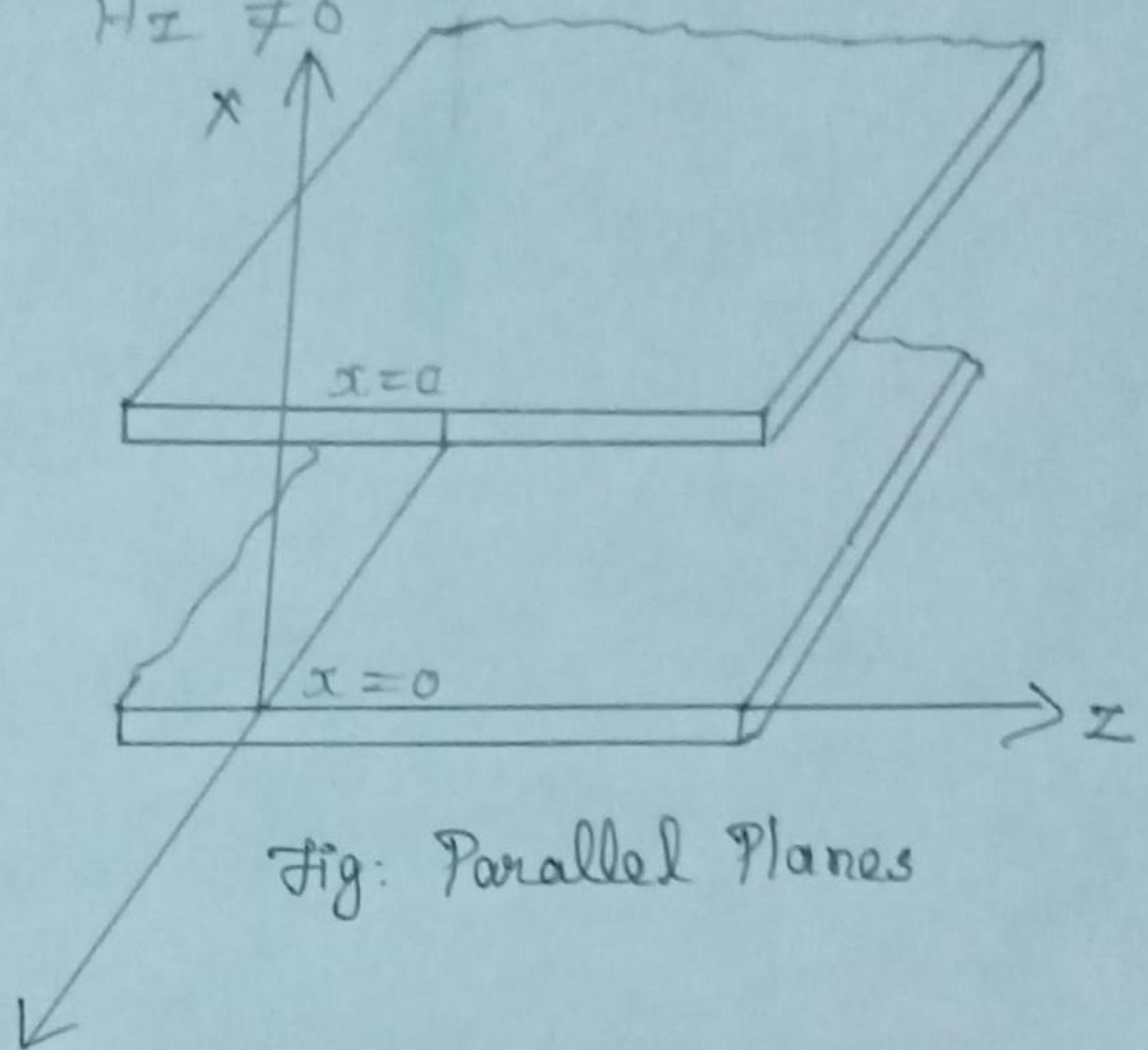


Fig: Parallel Planes

Here, $E_x = 0$, $H_y = 0$, $E_z = 0$

$\therefore$ The field components exists for TE waves in parallel planes are, E_y, H_x & H_z.

The wave equation for the component E_y is,

$$\frac{\partial^2 E_y}{\partial x^2} + \gamma^2 E_y = -\omega^2 \mu \varepsilon E_y \longrightarrow (7)$$

$$\frac{\partial^2 E_y}{\partial x^2} + \gamma^2 E_y + \omega^2 \mu \varepsilon E_y = 0$$

$$\frac{\partial^2 \underline{E_y}}{\partial x^2} + (\gamma^2 + \omega^2 \mu\epsilon)\underline{E_y} = 0$$

$$\frac{\partial^2 \underline{E_y}}{\partial x^2} + h^2 \underline{E_y} = 0 \longrightarrow (8)$$

This is a second order differential equation. The solution of this equation is,

$$\underline{E_y} = C_1 \sinh x + C_2 \cosh x \longrightarrow (9)$$

here, $\underline{E_y}$ - vector quantity

$\Rightarrow$ $\underline{E_y}$ is expressed in time and direction,

$$\underline{E_y} = E_y^{\circ} e^{-\gamma z} \longrightarrow (10)$$

Now the solution [equation (9)] becomes,

$$\underline{E_y} = [C_1 \sinh x + C_2 \cosh x] e^{-\gamma z} \longrightarrow (11)$$

C_1, C_2 - arbitrary constants

$\longrightarrow$ determined from boundary conditions

* The tangential component of 'E' is zero at the surface of conductor for all values of 'z'.

$\therefore$ $E_y = 0$ at $x = 0$ & $x = a$.

Boundary conditions are, $x = 0$ & $x = a$

i) $\underline{E_y} = 0$ when $x = 0$

$$\therefore \underline{E_y} = [C_1 \sin 0 + C_2 \cos 0] e^{-\gamma z} = 0 \qquad \left[\because \sin 0 = 0,\ \cos 0 = 1\right]$$

This is possible [ie., $E_y = 0$] only when $C_2 = 0$

Now the solution becomes [equ. (11)],

$$\underline{E_y} = [C_1 \sin hx + 0] e^{-\gamma z}$$

$$\therefore \quad E_y = C_1 \sin hx\, e^{-\gamma z} \longrightarrow (12)$$

ii) $E_y = 0$ when $x = a$

$$(12) \Rightarrow \therefore \quad E_y = C_1 \sin ha \cdot e^{-\gamma z} = 0$$

This is possible [ie., $E_y = 0$] only when $h = \frac{m\pi}{a}$.

$$\Rightarrow h = \frac{m\pi}{a}, \; m = 1, 2, 3, \cdots \qquad [\because \sin m\pi = 0]$$

now the solution becomes [equ. 12],

$$\boxed{E_y = C_1 \sin\left(\frac{m\pi}{a}\right) x \cdot e^{-\gamma z}} \longrightarrow (13)$$

we know that,

$$\gamma E_y = -j\omega\mu H_x$$

[refer general wave behaviour along uniform guiding structure]

$$\Rightarrow H_x = \frac{-\gamma}{j\omega\mu} E_y$$

$$\boxed{H_x = \frac{-\gamma}{j\omega\mu} C_1 \sin\left(\frac{m\pi}{a}\right) x \cdot e^{-\gamma z}} \longrightarrow (14)$$

Also we know that,

$$\frac{\partial E_y}{\partial x} = -j\omega\mu H_z$$

$$\Rightarrow H_z = \frac{-1}{j\omega\mu} \frac{\partial E_y}{\partial x}$$

$$= \frac{-1}{j\omega\mu} \frac{\partial}{\partial x}\left[C_1 \sin\left(\frac{m\pi}{a}\right) x \cdot e^{-\gamma z}\right]$$

$$\boxed{H_z = \frac{-m\pi}{j\omega\mu a} \cos\left(\frac{m\pi}{a}\right) x \cdot e^{-\gamma z}} \longrightarrow (15)$$

γ - propagation constant

where, $\gamma = \alpha + j\beta$

* If the wave propagates without attenuation, $\alpha = 0$, only 'β' exists

The field strengths for TE waves are,

$$\boxed{\begin{aligned} E_y &= C_1 \sin\left(\frac{m\pi}{a}\right)x \cdot e^{-j\beta z} \\ H_x &= \frac{\beta}{\omega\mu} C_1 \sin\left(\frac{m\pi}{a}\right)x \cdot e^{-j\beta z} \\ H_z &= -\frac{m\pi}{j\omega\mu a} C_1 \cos\left(\frac{m\pi}{a}\right)x \cdot e^{-j\beta z} \end{aligned}}$$

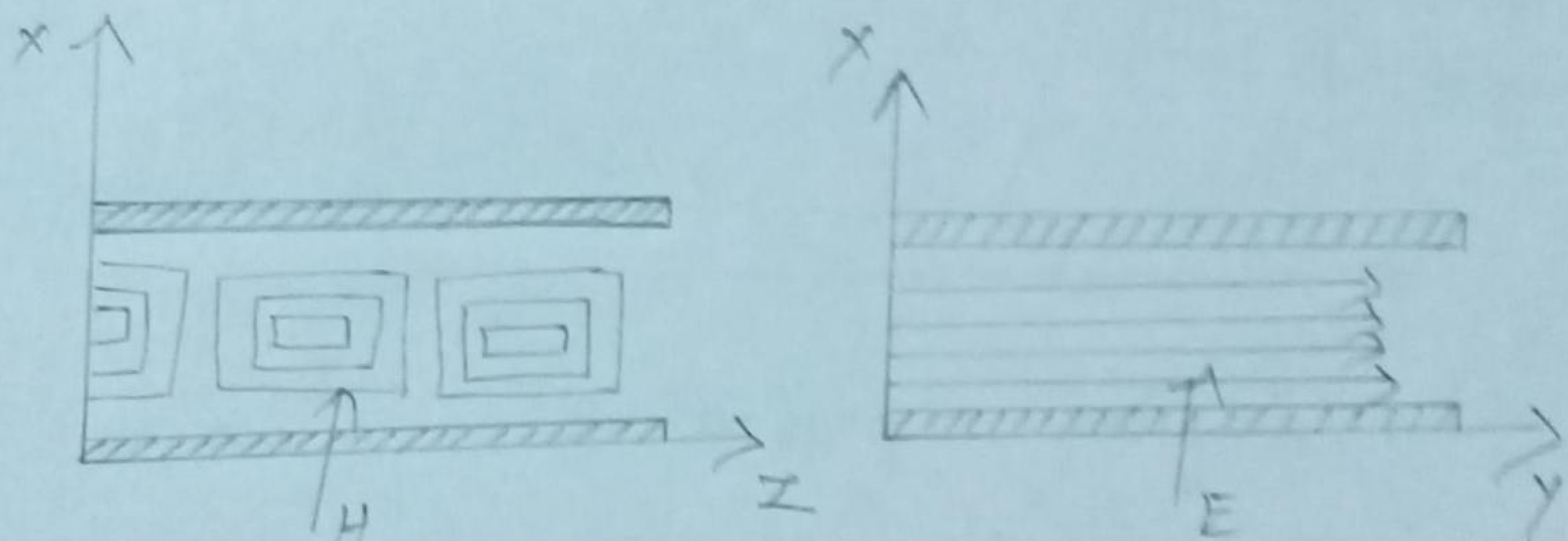
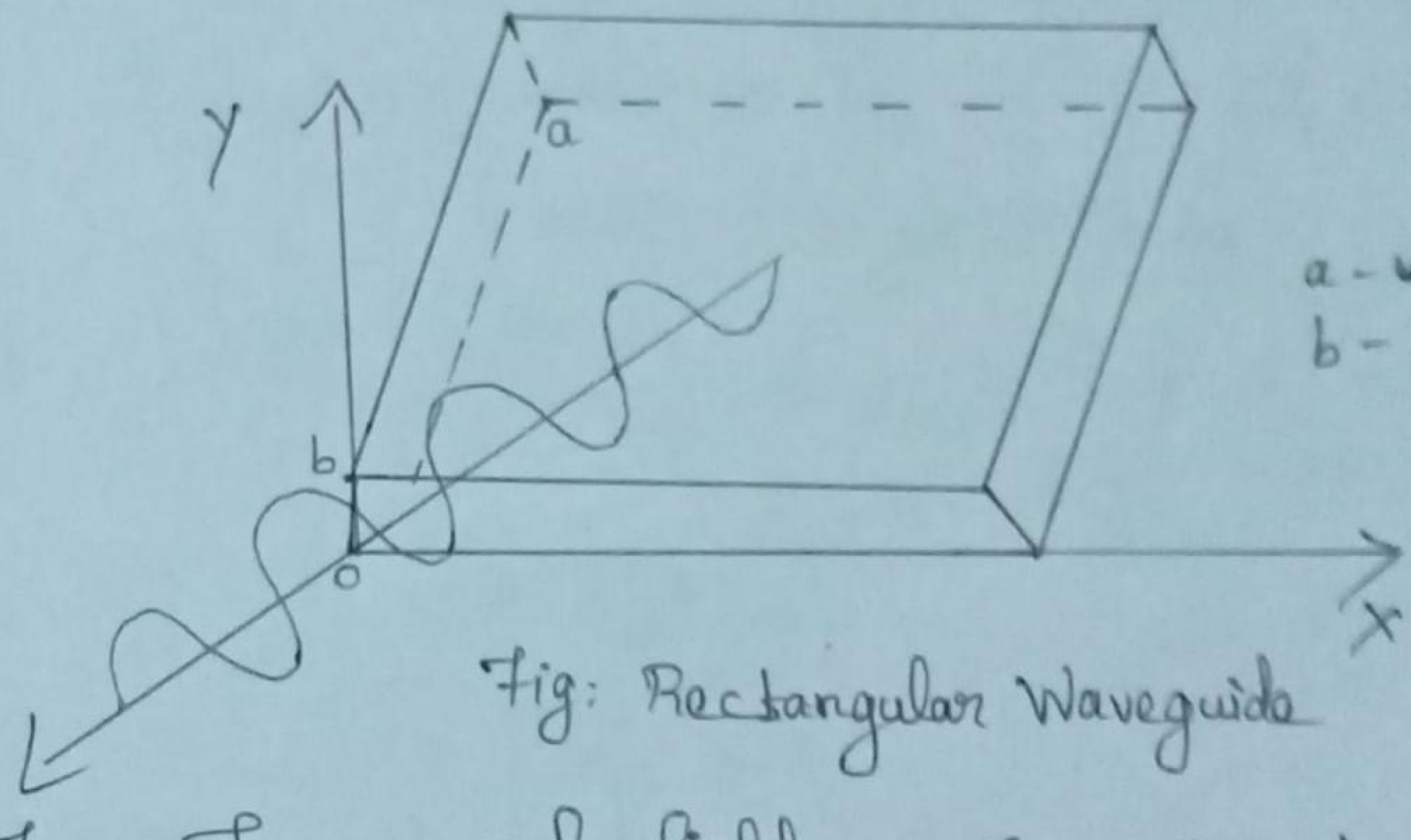


Fig: Electric and magnetic field between Parallel Planes

TM WAVES IN RECTANGULAR WAVEGUIDES

Transverse magnetic waves are the waves in which the magnetic field strength 'H' is entirely transverse. It has an electric field strength E_z in the direction of propagation and no component of magnetic field in the same direction.

ie., $H_z = 0$, $E_z \neq 0$



a - width
b - height

Fig: Rectangular Waveguide

The general field equations of rectangular waveguide is,

$$E_x = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y} \rightarrow (1)$$

$$E_y = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x} \rightarrow (2)$$

$$\Rightarrow \frac{1}{X}\frac{\partial^2 X}{\partial x^2} + h^2 = \frac{-1}{Y}\frac{\partial^2 Y}{\partial y^2} \longrightarrow (11)$$

[Function of 'x' alone] = [Function of 'y' alone]

$\therefore$ Expression in equation (11) can be equated to a constant.

i.e., $$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + h^2 = A^2 \longrightarrow (12)$$

$$\Rightarrow \frac{1}{X}\frac{\partial^2 X}{\partial x^2} + h^2 - A^2 = 0 \longrightarrow (13)$$

let, $B^2 = h^2 - A^2$

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + B^2 = 0 \longrightarrow (14)$$

Solution of equation (14) is, [Value of 'X']

$$X = C_1 \cos Bx + C_2 \sin Bx \longrightarrow (15)$$

similarly, $$\frac{-1}{Y}\frac{\partial^2 Y}{\partial y^2} = A^2 \longrightarrow (16)$$

$$-\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} - A^2 = 0$$

$$\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} + A^2 = 0 \longrightarrow (17)$$

Solution of equation (17) is, [Value of 'Y']

$$Y = C_3 \cos Ay + C_4 \sin Ay \longrightarrow (18)$$

wkt, $E_z^0 = XY$

Substitute the values of X and Y in E_z,

now, $E_z = [C_1 \cos Bx + C_2 \sin Bx][C_3 \cos Ay + C_4 \sin Ay]$

$\Rightarrow E_I^0 = C_1C_3\cos Bx\cos Ay + C_1C_4\cos Bx\sin Ay +$
$C_2C_3\sin Bx\cos Ay + C_2C_4\sin Bx\sin Ay \rightarrow (19)$

where,

C_1, C_2, C_3, C_4, A, B - Constants $\rightarrow$ determined by boundary conditions

Boundary conditions are,

$E_I^0 = 0$ when $x=0, x=a, y=0, y=b$

i) when $x=0$, $E_I^0 = 0$

$\therefore E_I^0 = C_1C_3\cos(B\times 0)\cos Ay + C_1C_4\cos(B\times 0)\sin Ay + C_2C_3\sin(B\times 0)\cos Ay + C_2C_4\sin(B\times 0)\sin Ay$

$\Rightarrow E_I^0 = C_1C_3\cos Ay + C_1C_4\sin Ay = 0 \rightarrow (20)$ $\left[\because \cos 0 = 1, \sin 0 = 0\right]$

This is possible only when $C_1 = 0$

Now the general solution is [equ(19)],

$E_I^0 = 0 + 0 + C_2C_3\sin Bx\cos Ay + C_2C_4\sin Bx\sin Ay$

$\Rightarrow E_I^0 = C_2C_3\sin Bx\cos Ay + C_2C_4\sin Bx\sin Ay \rightarrow (20)$

ii) when $y=0$, $E_I^0 = 0$

$(20) \Rightarrow E_I^0 = C_2C_3\sin Bx\cos(A\times 0) + C_2C_4\sin Bx\sin(A\times 0) = 0$

$\Rightarrow E_I^0 = C_2C_3\sin Bx = 0 \rightarrow (21)$

This is possible either $C_2=0$ or $C_3=0$.

If $C_2 = 0$, $E_I^0 = 0$ [equ(20)] $\therefore C_3 = 0$

$(20) \Rightarrow E_I^0 = 0 + C_2C_4\sin Bx\sin Ay$

Now the general solution is,

$E_I^0 = C_2C_4\sin Bx\sin Ay \rightarrow (22)$

$$H_x = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial x} + \frac{j\omega\epsilon}{h^2}\frac{\partial E_z}{\partial y} \rightarrow (3)$$

$$H_y = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\epsilon}{h^2}\frac{\partial E_z}{\partial x} \rightarrow (4)$$

The wave equation in rectangular waveguide is,

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \gamma^2 E_z = -\omega^2\mu\epsilon E_z \rightarrow (5)$$

The solution of the wave equation (5) is,

$$E_z(x, y, z) = E_z^0(x, y)e^{-\gamma z} \rightarrow (6)$$

Let, $E_z^0 = XY \rightarrow (7)$

where,

X - function of x alone

Y - function of y alone

Substituting the value of E_z in the wave equation (5)

$$\frac{\partial^2(XY)}{\partial x^2} + \frac{\partial^2(XY)}{\partial y^2} + \gamma^2 XY = -\omega^2\mu\epsilon XY$$

$$Y\frac{\partial^2 X}{\partial x^2} + X\frac{\partial^2 Y}{\partial y^2} + \gamma^2 XY + \omega^2\mu\epsilon XY = 0$$

$$Y\frac{\partial^2 X}{\partial x^2} + X\frac{\partial^2 Y}{\partial y^2} + (\gamma^2 + \omega^2\mu\epsilon)XY = 0 \rightarrow (8)$$

$$\Rightarrow Y\frac{\partial^2 X}{\partial x^2} + X\frac{\partial^2 Y}{\partial y^2} + h^2 XY = 0 \rightarrow (9)$$

where, $h^2 = \gamma^2 + \omega^2\mu\epsilon$

$\div$ de by XY,

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} + h^2 = 0 \rightarrow (10)$$

Using remaining two boundary conditions [$x=a$, $y=b$] calculate the value of constants 'A' and 'B'

iii) If $x=a$, $E_z^\circ = 0$

(22) $\Rightarrow \therefore \quad E_z^\circ = C_2C_4 \sin Ba \sin Ay = 0 \rightarrow (23)$

This is possible only if $B = \frac{m\pi}{a}$ for all values of 'y'. [$\because \sin m\pi = 0$, $m = 1, 2, 3 \ldots$]

$$\therefore \; B = \frac{m\pi}{a}, \quad m = 1, 2, 3, \ldots$$

now the general solution is,

$$E_z^\circ = C_2C_4 \sin \frac{m\pi}{a} x \sin Ay \rightarrow (24)$$

iv) If $y=b$, $E_z^\circ = 0$

(24) $\Rightarrow \quad \therefore E_z^\circ = C_2C_4 \sin \frac{m\pi}{a} x \sin Ab = 0 \rightarrow (25)$

This is possible only if $A = \frac{n\pi}{b}$ for all values of 'x'.

now the general solution is, $A = \frac{n\pi}{b}$, $n = 1, 2, 3, \ldots$

$$\therefore \; E_z = C_2C_4 \sin \frac{m\pi}{a} x \sin \frac{n\pi}{b} y \rightarrow (26)$$

$$\Rightarrow E_z^\circ = C_2C_4 \sin Bx \sin Ay \rightarrow (27)$$

(27) $\Rightarrow$

$$\frac{\partial E_z^\circ}{\partial x} = C_2C_4 \cos Bx \,.\, B \sin Ay$$

$$\Rightarrow \frac{\partial E_z^\circ}{\partial x} = C_2C_4 B \cos Bx \sin Ay$$

(27) $\Rightarrow$

$$\frac{\partial E_z^\circ}{\partial y} = \frac{\partial}{\partial y}\left[C_2C_4 \sin Bx \sin Ay\right]$$

$$= C_2C_4 \sin Bx \cos Ay \,.\, A$$

$$\frac{\partial E_z^\circ}{\partial y} = C_2C_4 A \sin Bx \cos Ay$$

To find the field equations of TM waves in rectangular waveguide substitute E_z and H_z values in equations (1), (2), (3) & (4).

$$(1) \Rightarrow E_x = \frac{-\gamma}{h^2} C_2 C_4 B \cos Bx \sin Ay - \frac{j\omega\mu}{h^2} \times 0 \quad (\because H_z = 0)$$

$$\Rightarrow E_x = \frac{-\gamma}{h^2} C_2 C_4 B \cos Bx \sin Ay \longrightarrow (28)$$

$$(2) \Rightarrow E_y = \frac{-\gamma}{h^2} C_2 C_4 A \sin Bx \cos Ay + \frac{j\omega\mu}{h^2} \times 0 \quad \left[\because H_z = 0,\ \frac{\partial H_z}{\partial y} = \frac{\partial H_z}{\partial x} = 0\right]$$

$$\Rightarrow E_y = \frac{-\gamma}{h^2} C_2 C_4 A \sin Bx \cos Ay \longrightarrow (29)$$

$$(3) \Rightarrow H_x = \frac{-\gamma}{h^2} \times 0 + \frac{j\omega\epsilon}{h^2} C_2 C_4 A \sin Bx \cos Ay$$

$$\Rightarrow H_x = \frac{j\omega\epsilon}{h^2} C_2 C_4 A \sin Bx \cos Ay \longrightarrow (30)$$

$$(4) \Rightarrow H_y = \frac{-\gamma}{h^2} \times 0 - \frac{j\omega\epsilon}{h^2} C_2 C_4 B \cos Bx \sin Ay$$

$$\Rightarrow H_y = \frac{-j\omega\epsilon}{h^2} C_2 C_4 B \cos Bx \sin Ay \longrightarrow (31)$$

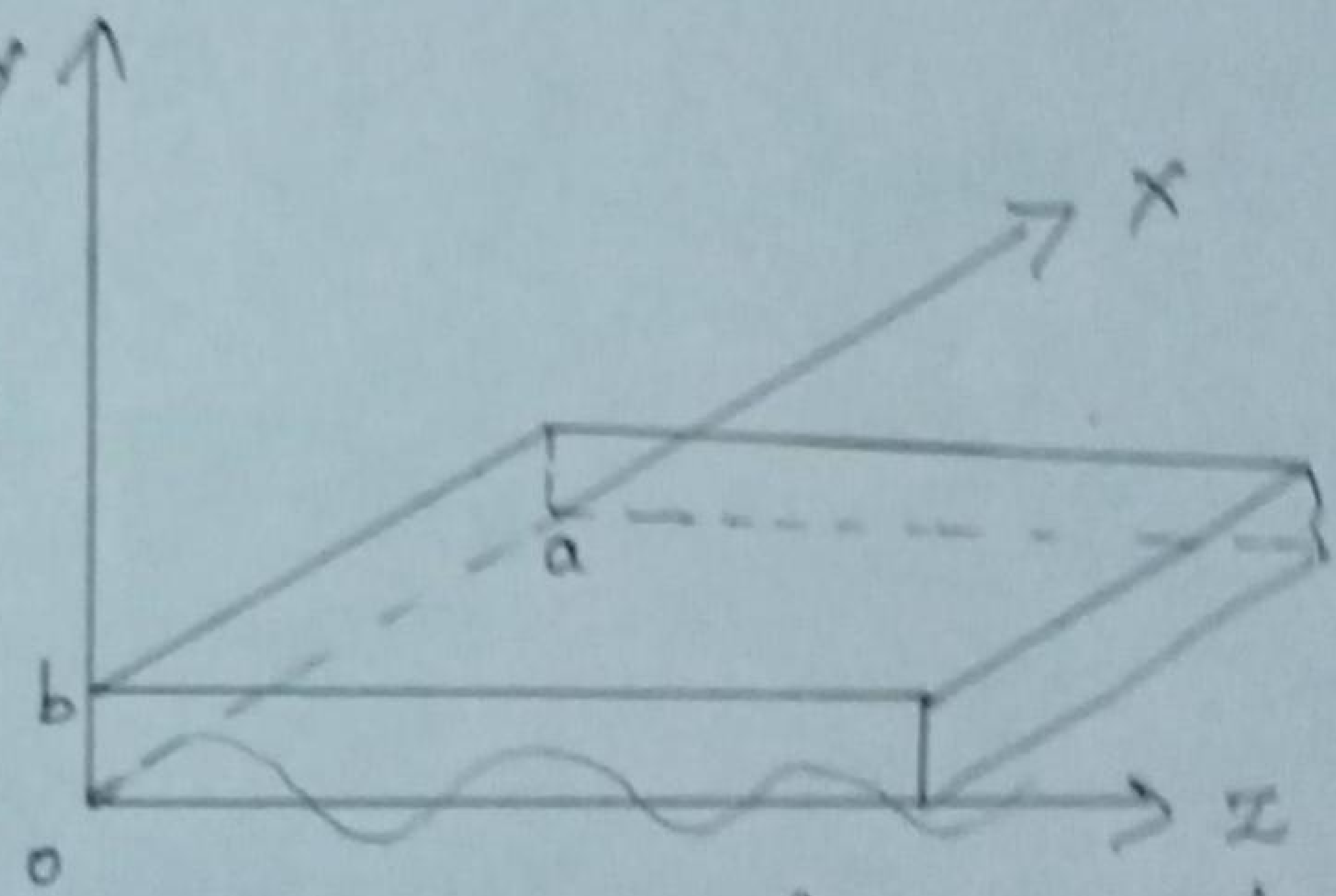
For propagation, $\gamma = j\beta$, $\alpha = 0$, the field equations of TM waves are,

$$\boxed{\begin{aligned} E_x &= \frac{-j\beta}{h^2} C_2 C_4 B \cos Bx \sin Ay \\ E_y &= \frac{-j\beta}{h^2} C_2 C_4 A \sin Bx \cos Ay \\ H_x &= \frac{j\omega\epsilon}{h^2} C_2 C_4 A \sin Bx \cos Ay \\ H_y &= \frac{-j\omega\epsilon}{h^2} C_2 C_4 B \cos Bx \sin Ay \end{aligned}}$$

TE WAVES IN RECTANGULAR WAVEGUIDE

Transverse electric waves (TE) are the waves in which the electric field strength is entirely transverse. It has a magnetic field strength in the direction of propagation and no component of electric field in the same direction.

i.e., $H_z \neq 0$, $E_z = 0$



a - width
b - height

Fig: Rectangular Waveguide

The general field equations of rectangular waveguide are,

$$E_x = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y} \rightarrow (1)$$

$$E_y = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x} \rightarrow (2)$$

$$H_x = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial x} + \frac{j\omega\epsilon}{h^2}\frac{\partial E_z}{\partial y} \rightarrow (3)$$

$$H_y = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\epsilon}{h^2}\frac{\partial E_z}{\partial x} \rightarrow (4)$$

The wave equation in a rectangular waveguide is

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \gamma^2 H_z = -\omega^2 \mu \varepsilon H_z \longrightarrow (5)$$

The solution of the wave equation is [value of H_z],

$$H_z(x, y, z) = H_z^0(x, y)\, e^{-\gamma z} \longrightarrow (6)$$

let, $H_z^0(x, y) = XY \longrightarrow (7)$

where,
X - function of 'x' alone
Y - function of 'y' alone

Substitute the value of H_z in wave equation (5),

$$\frac{\partial^2 XY}{\partial x^2} + \frac{\partial^2 XY}{\partial y^2} + \gamma^2 XY = -\omega^2 \mu \varepsilon XY$$

$$Y\frac{\partial^2 X}{\partial x^2} + X\frac{\partial^2 Y}{\partial y^2} + \gamma^2 XY + \omega^2 \mu \varepsilon XY = 0$$

$$Y\frac{\partial^2 X}{\partial x^2} + X\frac{\partial^2 Y}{\partial y^2} + (\gamma^2 + \omega^2 \mu \varepsilon)XY = 0$$

$$Y\frac{\partial^2 X}{\partial x^2} + X\frac{\partial^2 Y}{\partial y^2} + h^2 XY = 0 \longrightarrow (8)$$

where, $h^2 = \gamma^2 + \omega^2 \mu \varepsilon$

Dividing by XY,

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} + h^2 = 0 \longrightarrow (9)$$

$$\underbrace{\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + h^2}_{\text{Function of 'x' alone}} = \underbrace{-\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2}}_{\text{Function of 'y' alone}} \longrightarrow (10)$$

$\therefore$ Equation (10) can be equated to a constant.

i.e., $\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + h^2 = A^2$

$$\Rightarrow \frac{1}{X}\frac{\partial^2 X}{\partial x^2} + h^2 - A^2 = 0$$

$$\Rightarrow \frac{1}{X}\frac{\partial^2 X}{\partial x^2} + B^2 = 0 \longrightarrow (11)$$

where, $B^2 = h^2 - A^2$

The solution of equation (11) is [value of 'X'],

$$X = C_1 \cos Bx + C_2 \sin Bx \longrightarrow (12)$$

Similarly, $-\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} = A^2$

$$\Rightarrow \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} + A^2 = 0 \longrightarrow (13)$$

The solution of equation (13) [value of 'Y'] is,

$$Y = C_3 \cos Ay + C_4 \sin Ay \longrightarrow (14)$$

now, $H_z^0 = XY$

$$= [C_1 \cos Bx + C_2 \sin Bx][C_3 \cos Ay + C_4 \sin Ay]$$

$$= C_1C_3 \cos Bx \cos Ay + C_1C_4 \cos Bx \sin Ay + C_2C_3 \sin Bx \cos Ay + C_2C_4 \sin Bx \sin Ay \longrightarrow (15)$$

wkt,

$$(1) \Rightarrow E_x = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$$

For TE waves, $E_z = 0$

$$\therefore E_x = \frac{-\gamma}{h^2} \times 0 - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$$

$$\Rightarrow E_x = \frac{-j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$$

$$= \frac{-j\omega\mu}{h^2}\frac{\partial}{\partial y}[C_1C_3 \cos Bx \cos Ay + C_1C_4 \cos Bx \sin Ay + C_2C_3 \sin Bx \cos Ay + C_2C_4 \sin Bx \sin Ay]$$

$$E_x = \frac{-j\omega\mu}{h^2}\Big[-C_1C_3A\sin Ay\cos Bx + C_1C_4A\cos Bx\cos Ay - C_2C_3A\sin Bx\sin Ay + C_2C_4A\sin Bx\cos Ay\Big] \rightarrow (16)$$

* To find the general solution of E_x apply the boundary conditions.

Boundary conditions are,

$E_x = 0$ when $y=0$, $y=b$

i) If $y=0$, $E_x = 0$

$$(16) \Rightarrow E_x = \frac{-j\omega\mu}{h^2}\Big[-C_1C_3A\sin(A\times 0)\cos Bx + C_1C_4A\cos Bx\cos(A\times 0) - C_2C_3A\sin Bx\sin(A\times 0) + C_2C_4A\sin Bx\cos(A\times 0)\Big] = 0$$

$$E_x = \frac{-j\omega\mu}{h^2}\Big[C_1C_4A\cos Bx + C_2C_4A\sin Bx\Big] = 0 \quad \Big[\because \sin 0 = 0,\ \cos 0 = 1\Big]$$

This is possible only if $C_4 = 0$ [$\because C_4$ is common]

Then the general solution [equ. 16] is,

$$E_x = \frac{-j\omega\mu}{h^2}\Big[-C_1C_3A\sin Ay\cos Bx + 0 - C_2C_3A\sin Bx\sin Ay + 0\Big]$$

$$E_x = \frac{j\omega\mu}{h^2}\Big[C_1C_3A\sin Ay\cos Bx + C_2C_3A\sin Bx\sin Ay\Big] \rightarrow (17)$$

ii) If $y=b$, $E_x = 0$

$$(17) \Rightarrow E_x = \frac{j\omega\mu}{h^2}\Big[C_1C_3A\sin Ab\cos Bx + C_2C_3A\sin Bx\sin Ab\Big] = 0$$

This is possible only if $A = \frac{n\pi}{b}$ $\quad$ [$\because \sin(n\pi) = 0$]

Then the general solution is, (equ. 17),

$$E_x = \frac{j\omega\mu}{h^2}\left[C_1C_3A\sin Ay\cos Bx + C_2C_3A\sin Ay\sin Bx\right] \quad \rightarrow (18)$$

Similarly for E_y,

$$(2) \Rightarrow \quad E_y = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}$$

For TE waves, $E_z = 0$

$$\therefore E_y = \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}$$

$$\Rightarrow E_y = \frac{j\omega\mu}{h^2}\frac{\partial}{\partial x}\left[C_1C_3\cos Bx\cos Ay + C_1C_4\cos Bx\sin Ay + C_2C_3\sin Bx\cos Ay + C_2C_4\sin Bx\sin Ay\right]$$

$$= \frac{j\omega\mu}{h^2}\left[C_1C_3B\sin Bx\cos Ay - C_1C_4\sin Bx\sin Ay + C_2C_3B\cos Bx\cos Ay + C_2C_4B\cos Bx\sin Ay\right] \quad \rightarrow (19)$$

* To find the general solution of E_y apply the boundary conditions.

Boundary conditions are,

$E_y = 0$, when $x = 0$, $x = a$

i) If $x = 0$, $E_y = 0$ $\quad [\because \sin 0 = 0$

(19) $\Rightarrow$

$$E_y = \frac{j\omega\mu}{h^2}\left[-C_1C_3B\sin(B\times 0)\cos Ay - C_1C_4\sin(B\times 0)\sin Ay + C_2C_3B\cos(B\times 0)\cos Ay + C_2C_4B\cos(B\times 0)\sin Ay\right] = 0$$

$$\Rightarrow E_y = \frac{j\omega\mu}{h^2}\left[C_2C_3B\cos Ay + C_2C_4B\sin Ay\right] = 0$$

This is possible only if $C_2 = 0$

Then the general solution (eqn. 19) is,

$$E_y = \frac{j\omega\mu}{h^2}\left[-C_1C_3B\sin Bx\cos Ay - C_1C_4\sin Bx\sin Ay + 0 + 0\right]$$

$$E_y = \frac{j\omega\mu}{h^2}\left[-C_1C_3B\sin Bx\cos Ay - C_1C_4\sin Bx\sin Ay\right] \rightarrow (20)$$

ii) If $x = a$ then $E_y = 0$

(20) $\Rightarrow$ $$E_y = \frac{j\omega\mu}{h^2}\left[-C_1C_3B\sin Ba\cos Ay - C_1C_4\sin Ba\sin Ay\right] = 0$$

This is possible only when $B = \frac{m\pi}{a}$ $\left[\because \sin m\pi = 0\right]$

Then the general solution is,

$$E_y = \frac{-j\omega\mu}{h^2}\left[C_1C_3B\sin Bx\cos Ay - C_1C_4\sin Bx\sin Ay\right] \rightarrow (21)$$

Substituting the values $C_2 = C_4 = 0$ &

$A = \frac{n\pi}{b}$, $B = \frac{m\pi}{a}$ in equations (18) and (21)

(18) $\Rightarrow$ $$E_x = \frac{j\omega\mu}{h^2}\left[C_1C_3A\sin\left(\frac{n\pi}{b}\right)y\cos\left(\frac{m\pi}{a}\right)x + 0\right]$$

$$E_x = \frac{j\omega\mu}{h^2}\left[C_1C_3A\cos\left(\frac{m\pi}{a}\right)x\sin\left(\frac{n\pi}{b}\right)y\right] \rightarrow (22)$$

(21) $\Rightarrow$ $$E_y = \frac{-j\omega\mu}{h^2}\left[C_1C_3B\sin\left(\frac{m\pi}{a}\right)x.\cos\left(\frac{n\pi}{b}\right)y - 0\right]$$

$$E_y = \frac{-j\omega\mu}{h^2}\left[C_1C_3B\sin\left(\frac{m\pi}{a}\right)x.\cos\left(\frac{n\pi}{b}\right)y\right] \rightarrow (23)$$

H_x , $\underline{H_y}$

$\underline{\text{III}^{ly}}$ for H_x,

$$(3) \Rightarrow \quad H_x = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial x} + \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial y}$$

For TE waves $E_z = 0$,

$$\therefore \quad H_x = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial x}$$

But, $\underline{E_y} = \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}$

$$\Rightarrow \quad \frac{\partial H_z}{\partial x} = \frac{h^2}{j\omega\mu}\cdot \underline{E_y}$$

$$= \frac{\cancel{h^2}}{\cancel{j\omega\mu}}\cdot\frac{-\cancel{j\omega\mu}}{\cancel{h^2}} C_1C_3B\sin\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y$$

$$\Rightarrow \quad \frac{\partial H_z}{\partial x} = -C_1C_3B\sin\left(\frac{m\pi}{a}\right)x\cos\left(\frac{n\pi}{b}\right)y$$

now, $H_x = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial x} = \frac{-\gamma}{h^2}\left[-C_1C_3B\sin\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y\right]$

$$H_x = \frac{\gamma}{h^2}C_1C_3B\sin\left(\frac{m\pi}{a}\right)x\cos\left(\frac{n\pi}{b}\right)y \longrightarrow (24)$$

$\underline{\text{III}^{ly}}$ for $\underline{H_y}$,

$$(4) \Rightarrow \quad \underline{H_y} = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial y} - \frac{j\omega.\varepsilon}{h^2}\frac{\partial E_z}{\partial x}$$

For TE waves , $E_z = 0$

$$\therefore \quad H_y = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial y}$$

But, $E_x = \frac{-j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$

$$\Rightarrow \frac{\partial H_z}{\partial y} = -\frac{h^2}{j\omega\mu} E_x$$

$$= -\frac{\cancel{h^2}}{\cancel{j\omega\mu}} \cdot \frac{\cancel{j\omega\mu}}{\cancel{h^2}} C_1 C_3 A \cos\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y$$

$$\frac{\partial H_z}{\partial y} = -C_1 C_3 A \cos\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y$$

now, $H_y = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial y} = \frac{-\gamma}{h^2}\left[-C_1 C_3 A \cos\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y\right]$

$$\Rightarrow H_y = \frac{\gamma}{h^2} C_1 C_3 A \cos\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y \rightarrow (25)$$

The field equations of TE waves in rectangular waveguide are,

$$E_x = \frac{j\omega\mu}{h^2} C_1 C_3 A \cos\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y$$

$$E_y = \frac{-j\omega\mu}{h^2} C_1 C_3 B \sin\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y$$

$$H_x = \frac{\gamma}{h^2} C_1 C_3 B \sin\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y$$

$$H_y = \frac{-\gamma}{h^2} C_1 C_3 A \cos\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y$$

$$H_z = C_1 C_3 \cos\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y$$

BESSEL FUNCTIONS

Bessel functions, first defined by the mathematician Daniel Bernoulli and then generalized by Friedrich Bessel, are canonical solutions $y(x)$ of Bessel's differential equation for an arbitrary complex number α, the order of the Bessel function.

Bessel functions are used to solve the wave equation at a given frequency. In solving for the electromagnetic fields within the circular waveguides, a differential equation known as Bessel's equation is encountered. The solution of the equation leads to Bessel functions.

The differential equation is,

$$\frac{d^2P}{d\rho^2} + \frac{1}{\rho}\frac{dP}{d\rho} + \left(1 - \frac{n^2}{\rho^2}\right)P = 0, \quad n = 0, 1, 2, \ldots \rightarrow (1)$$

The solution of this Bessel's equation can be obtained by assuming a power series expansion.

$$P = a_0 + a_1\rho + a_2\rho^2 + a_3\rho^3 + \ldots \rightarrow (2)$$

For special case, $n=0$, the Bessel's equation becomes,

$$\frac{d^2P}{d\rho^2} + \frac{1}{\rho}\frac{dP}{d\rho} + P = 0 \longrightarrow (*)$$

Substituting the value of 'P' in equation (*) and equating the sums of the co-efficients of each power of ρ to zero.

$$\therefore P = P_1 = c_1\left[1 - \left(\frac{\rho}{2}\right)^2 + \frac{(\frac{1}{2}\rho)^4}{(2!)^2} - \frac{(\frac{1}{2}\rho)^6}{(3!)^2} + \dots\right]$$

$$\Rightarrow P = P_1 = c_1 \sum_{r=0}^{\infty} (-1)^n \frac{(\frac{1}{2})^2}{(r!)^2}$$

The series is convergent for all values of ρ either real or complex. This is called Bessel's function of the first kind of order zero and is denoted by $J_0(\rho) = P_0$ for $n=0$.

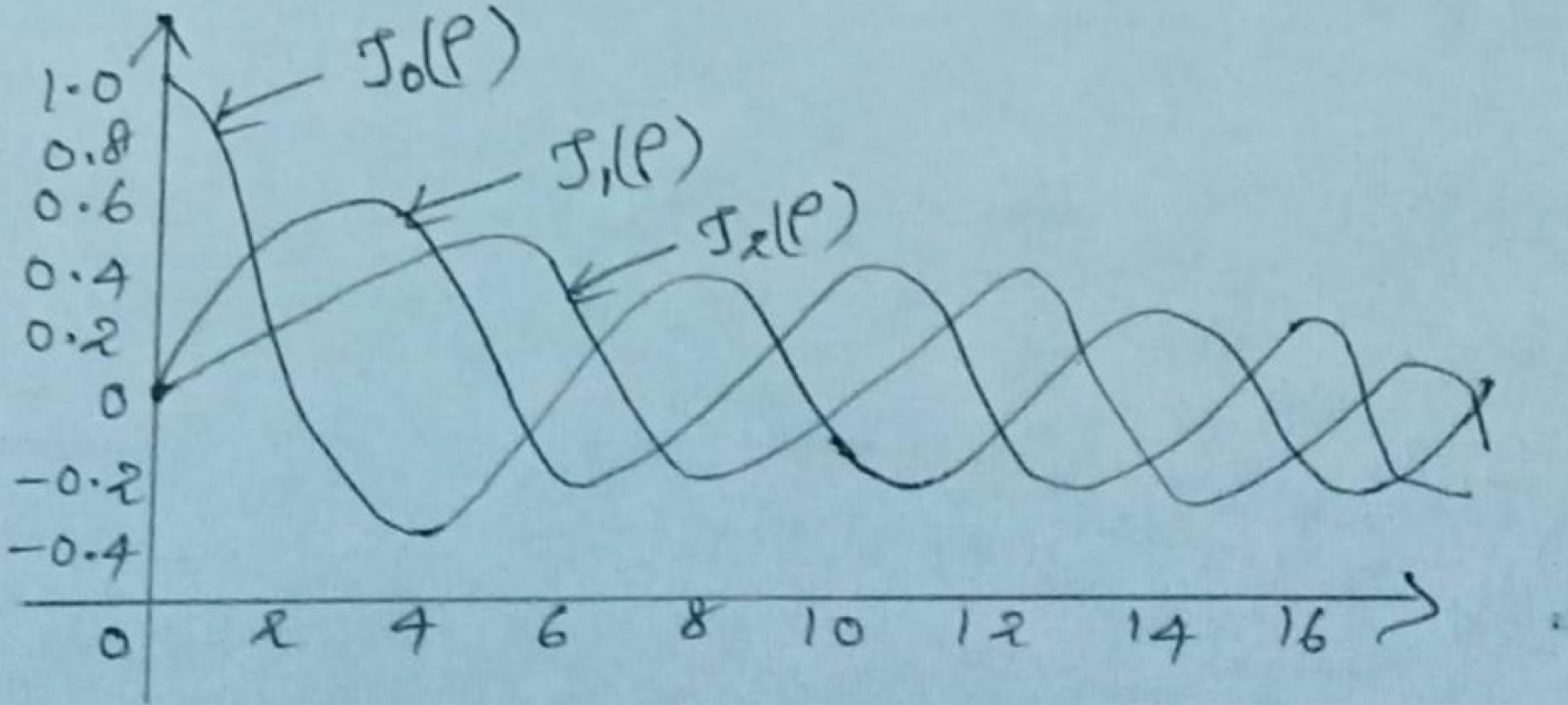


Fig: Bessel Functions of First Kind of Different Orders

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TM WAVES IN CIRCULAR WAVEGUIDE

Transverse magnetic waves are waves in which the magnetic field strength is entirely transverse. It has an electric field strength in the direction of propagation and no component of magnetic field strength in the same direction.

i.e., $H_z = 0$, $E_z \neq 0$

The boundary conditions require that E_z must vanish at the surface of the guide.

$$\therefore J_n(ha) = 0$$

where,

a - radius of the circular waveguide

The general field equations of circular waveguide are,

$$h^2 H_\rho = \frac{j\omega\varepsilon}{\rho}\frac{\partial E_z}{\partial \phi} - \gamma\frac{\partial H_z}{\partial \rho} \longrightarrow (1)$$

$$h^2 H_\phi = -j\omega\varepsilon\frac{\partial E_z}{\partial \rho} - \frac{\gamma}{\rho}\frac{\partial H_z}{\partial \phi} \longrightarrow (2)$$

$$h^2 E_\rho = -\gamma\frac{\partial E_z}{\partial \rho} - j\frac{\omega\mu}{\rho}\frac{\partial H_z}{\partial \phi} \longrightarrow (3)$$

$$h^2 E_\phi = \frac{-\gamma}{\rho}\frac{\partial E_z}{\partial \phi} + j\omega\mu\frac{\partial H_z}{\partial \rho} \longrightarrow (4)$$

Substitute $H_z = 0$

now The field equations of TM waves in circular waveguide is,

$$h^2 H_\rho = \frac{j\omega\varepsilon}{\rho}\frac{\partial E_z}{\partial \phi} \longrightarrow (5)$$

$$h^2 H_\phi = -j\omega\varepsilon \frac{\partial E_z}{\partial \rho} \rightarrow (6)$$

$$h^2 E_\rho = -\gamma \frac{\partial E_z}{\partial \rho} \rightarrow (7)$$

$$h^2 E_\phi = \frac{-\gamma}{\rho} \frac{\partial E_z}{\partial \phi} \rightarrow (8)$$

The expression for E_z for TM wave is,

$$E_z^0 = A_n J_n(h\rho) \cos n\phi$$

Differentiate with respect to 'ρ',

$$\frac{\partial E_z^0}{\partial \rho} = \frac{\partial}{\partial \rho}[A_n J_n(h\rho) \cos n\phi]$$

$$\Rightarrow \frac{\partial E_z^0}{\partial \rho} = A_n \frac{\partial J_n(h\rho)}{\partial \rho} \cos n\phi \rightarrow (9)$$

Differentiate E_z with respect to 'ϕ',

$$\frac{\partial E_z^0}{\partial \phi} = \frac{\partial}{\partial \phi}[A_n J_n(h\rho) \cos n\phi]$$

$$= A_n J_n(h\rho) - \sin(n\phi) \cdot n$$

$$\frac{\partial E_z^0}{\partial \phi} = -n A_n J_n(h\rho) \sin n\phi \rightarrow (10)$$

Substitute the value of $\frac{\partial E_z^0}{\partial \phi}$, $\frac{\partial E_z^0}{\partial \rho}$ in equations (5), (6), (7) & (8).

$$(5) \Rightarrow \quad H_\rho = \frac{-j\omega\varepsilon n}{\rho h^2} A_n J_n(h\rho) \sin n\phi \rightarrow (11)$$

$$(6) \Rightarrow \quad H_\phi = \frac{-j\omega\varepsilon}{h^2} A_n \frac{\partial [J_n(h\rho)]}{\partial \rho} \cos n\phi \rightarrow (12)$$

(7) $\Rightarrow$ $$E_\rho = \frac{-\gamma}{h^2} A_n \frac{\partial[J_n(h\rho)]}{\partial\rho} \cos n\phi \rightarrow (13)$$

(8) $\Rightarrow$ $$E_\phi = \frac{\gamma n}{h^2\rho} A_n J_n(h\rho) \sin n\phi \rightarrow (14)$$

If the wave propagates without attenuation $\alpha = 0$, only β exists.

The field equations of TM waves in circular waveguide is,

$$\boxed{\begin{aligned} H_\rho &= \frac{-j\omega\epsilon n A_n J_n(h\rho) \sin n\phi}{\rho h^2} \\ H_\phi &= \frac{-j\omega\epsilon}{h^2} A_n \frac{\partial[J_n(h\rho)]}{\partial\rho} \cos n\phi \\ E_\rho &= \frac{-j\beta}{h^2} A_n \frac{\partial[J_n(h\rho)]}{\partial\rho} \cos n\phi \\ E_\phi &= \frac{j\beta n}{h^2\rho} A_n J_n(h\rho) \sin n\phi \end{aligned}}$$

wkt, $J_n(ha) = 0$

The few roots are,

$(ha)_{01} = 2.405$

$(ha)_{11} = 3.85$

$(ha)_{02} = 5.52$

$(ha)_{12} = 7.02$

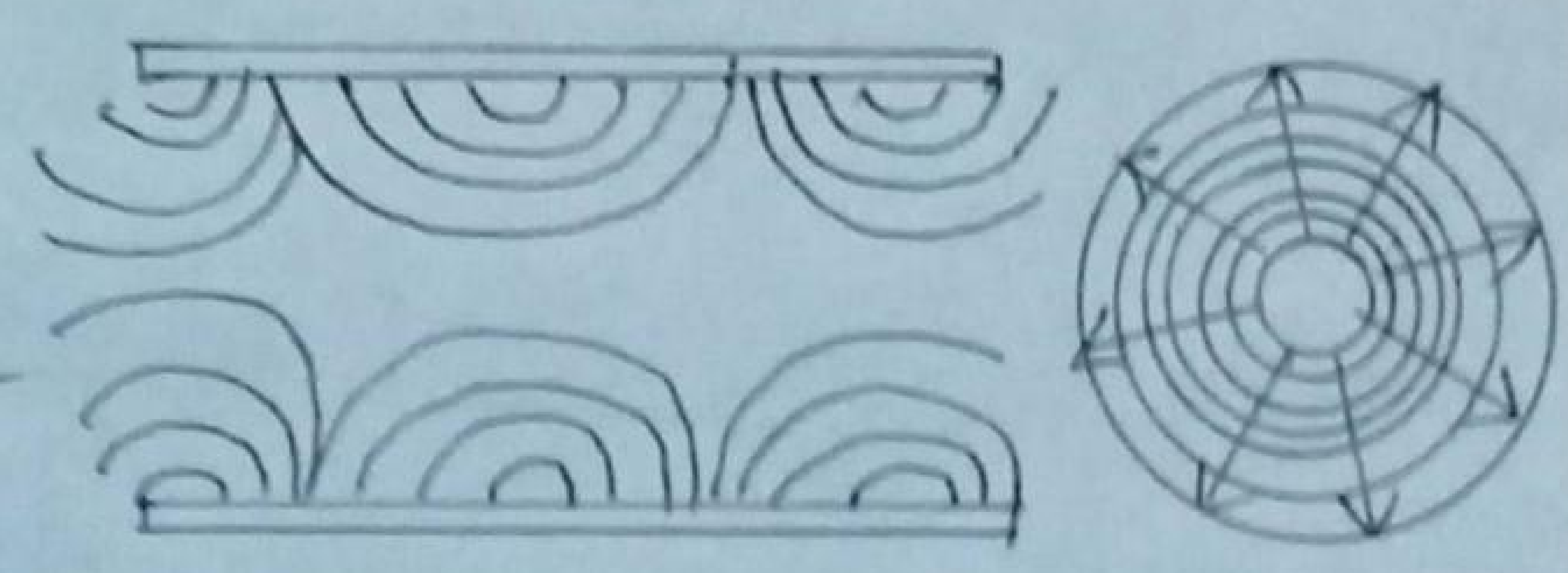

Fig: TM waves in Circular Waveguide

TE WAVES IN CIRCULAR WAVEGUIDE

Transverse electric waves are the waves in which the electric field strength 'E' is entirely transverse. It has a magnetic field strength in the direction of propagation and no component of electric field in the same direction.

i.e., $H_z \neq 0$, $E_z = 0$

For transverse electric waves, E_z is identically zero. The field equation of circular waveguide are,

$$h^2 H_\rho = \frac{j\omega\varepsilon}{\rho}\frac{\partial E_z}{\partial \phi} - \gamma\frac{\partial H_z}{\partial \rho} \rightarrow (1)$$

$$h^2 H_\phi = -j\omega\varepsilon\frac{\partial E_z}{\partial \rho} - \frac{\gamma}{\rho}\frac{\partial H_z}{\partial \phi} \rightarrow (2)$$

$$h^2 E_\rho = -\gamma\frac{\partial E_z}{\partial \rho} - \frac{j\omega\mu}{\rho}\frac{\partial H_z}{\partial \phi} \rightarrow (3)$$

$$h^2 E_\phi = -\frac{\gamma}{\rho}\frac{\partial E_z}{\partial \phi} + j\omega\mu\frac{\partial H_z}{\partial \rho} \rightarrow (4)$$

For TE waves, $E_z = 0$

now equations (1), (2), (3) & (4) becomes,

$$h^2 H_\rho = -\gamma \frac{\partial H_z}{\partial \rho} \longrightarrow (5)$$

$$h^2 H_\phi = -\frac{\gamma}{\rho} \frac{\partial H_z}{\partial \phi} \longrightarrow (6)$$

$$h^2 E_\rho = -\frac{j\omega\mu}{\rho} \frac{\partial H_z}{\partial \phi} \longrightarrow (7)$$

$$h^2 E_\phi = j\omega\mu \frac{\partial H_z}{\partial \rho} \longrightarrow (8)$$

The expression of H_z for TE wave is,

$$H_z^0 = C_n J_n(\rho h) \cos n\phi \longrightarrow (9)$$

Differentiate H_z^0 with respect to 'ρ',

$$\frac{\partial H_z^0}{\partial \rho} = \cancel{C_n \frac{J_n(\rho h) . h \cos n\phi}{\partial \rho}}$$

$$= C_n \cos n\phi \frac{\partial(\rho h)}{\partial \rho}$$

$$= C_n \cos n\phi \frac{\partial(\rho h)}{\partial \rho} . h$$

$$\Rightarrow \frac{\partial H_z^0}{\partial \rho} = C_n h \frac{\partial(\rho h)}{\partial \rho} \cos n\phi \longrightarrow (10)$$

Differentiate H_z^0 with respect to 'ϕ',

$$\frac{\partial H_z^0}{\partial \phi} = \frac{\partial}{\partial \phi} \left[C_n J_n(\rho h) \cos n\phi \right]$$

$$\frac{\partial H_z^\circ}{\partial \phi} = -C_n n J_n(\rho h) \sin n\phi \longrightarrow (11)$$

Substitute equ. (10), (11) in equation (5), (6), (7) & (8),

$$(5) \Rightarrow \quad H_\rho = \frac{-\gamma}{h^2} C_n h \frac{\partial(\rho h)}{\partial \rho} \cos n\phi \longrightarrow (12)$$

$$(6) \Rightarrow \quad H_\phi = \frac{\gamma}{\rho h^2} C_n \cdot n \cdot J_n(\rho h) \sin n\phi \longrightarrow (13)$$

$$(7) \Rightarrow \quad E_\rho = \frac{j\omega\mu}{\rho h^2} C_n \cdot n \cdot J_n(\rho h) \sin n\phi \longrightarrow (14)$$

$$(8) \Rightarrow \quad E_\phi = \frac{j\omega\mu}{h^2} C_n h \frac{\partial(\rho h)}{\partial \rho} \cos n\phi \longrightarrow (15)$$

If the wave propagates without attenuation, $\alpha = 0$, only 'β' exists

The field strengths of TE waves in circular waveguide are,

$$\boxed{\begin{aligned} H_\rho &= \frac{-j\beta}{h^2} C_n \cdot h \cdot \frac{\partial(\rho h)}{\partial \rho} \cos n\phi \\ H_\phi &= \frac{j\beta}{\rho h^2} C_n \cdot n \cdot J_n(\rho h) \sin n\phi \\ E_\rho &= \frac{j\omega\mu}{\rho h^2} C_n \cdot n J_n(\rho h) \sin n\phi \\ E_\phi &= \frac{j\omega\mu}{h^2} C_n h \frac{\partial(\rho h)}{\partial \rho} \cos n\phi \end{aligned}}$$

The boundary condition is $E_\phi = 0$ at $\rho = a$.

Since E_ϕ is proportional to $\frac{\partial H_z}{\partial \rho}$, $\frac{\partial J_n(ha)}{\partial \rho} = 0$

$$J_n'(ha) = 0$$

The first few roots are,

$$(ha)'_{01} = 3.83$$

$$(ha)'_{11} = 1.84$$

$$(ha)'_{02} = 7.02$$

$$(ha)'_{12} = 5.33$$

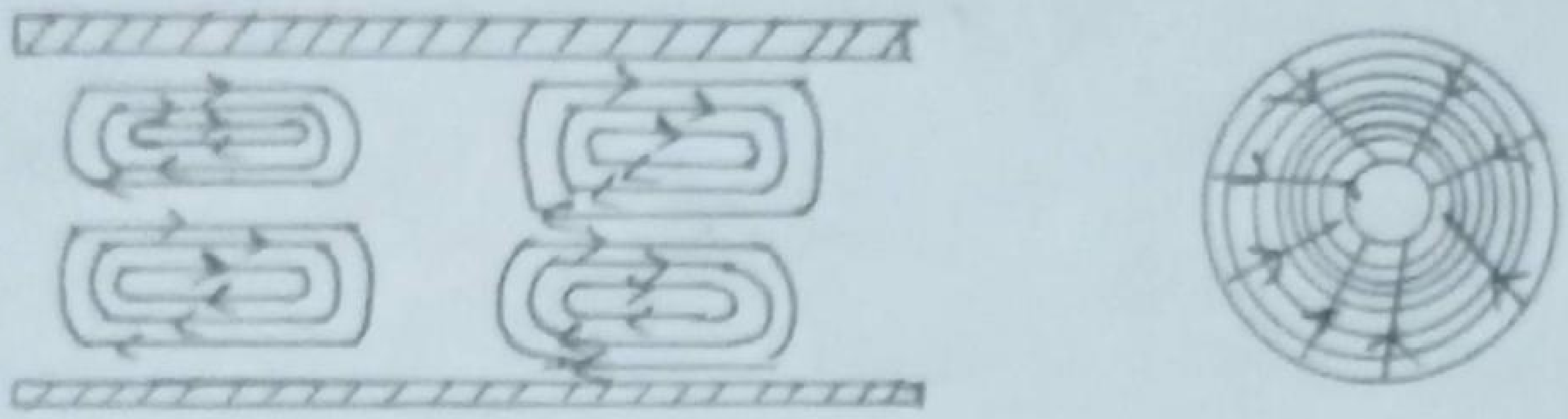

Fig: TE Waves in Circular Waveguide

UNIT-V

RF SYSTEM DESIGN CONCEPTS

Active RF components: Semiconductor basiss in RF, Bipolar Junction Transistors, RF field effect transistors, High electron mobility transistors, Basic concepts of RF design; Mixers, Low noise amplifiers, voltage control oscillators, Power amplifiers, Transducer power gain and stability considerations.

ACTIVE RF COMPONENTS

* Power Amplifiers
* Low Noise Amplifiers
* Mixers
* Oscillators

Semiconductor Basics in RF

Semiconductors are materials which have a conductivity between conductors and insulators [non conductors].

Ex: Germanium (Ge), Silicon (Si), Gallium Arsenide (GaAs)

* The operation of semiconductor devices is naturally dependent on the physical behavior of the semiconductors themselves.

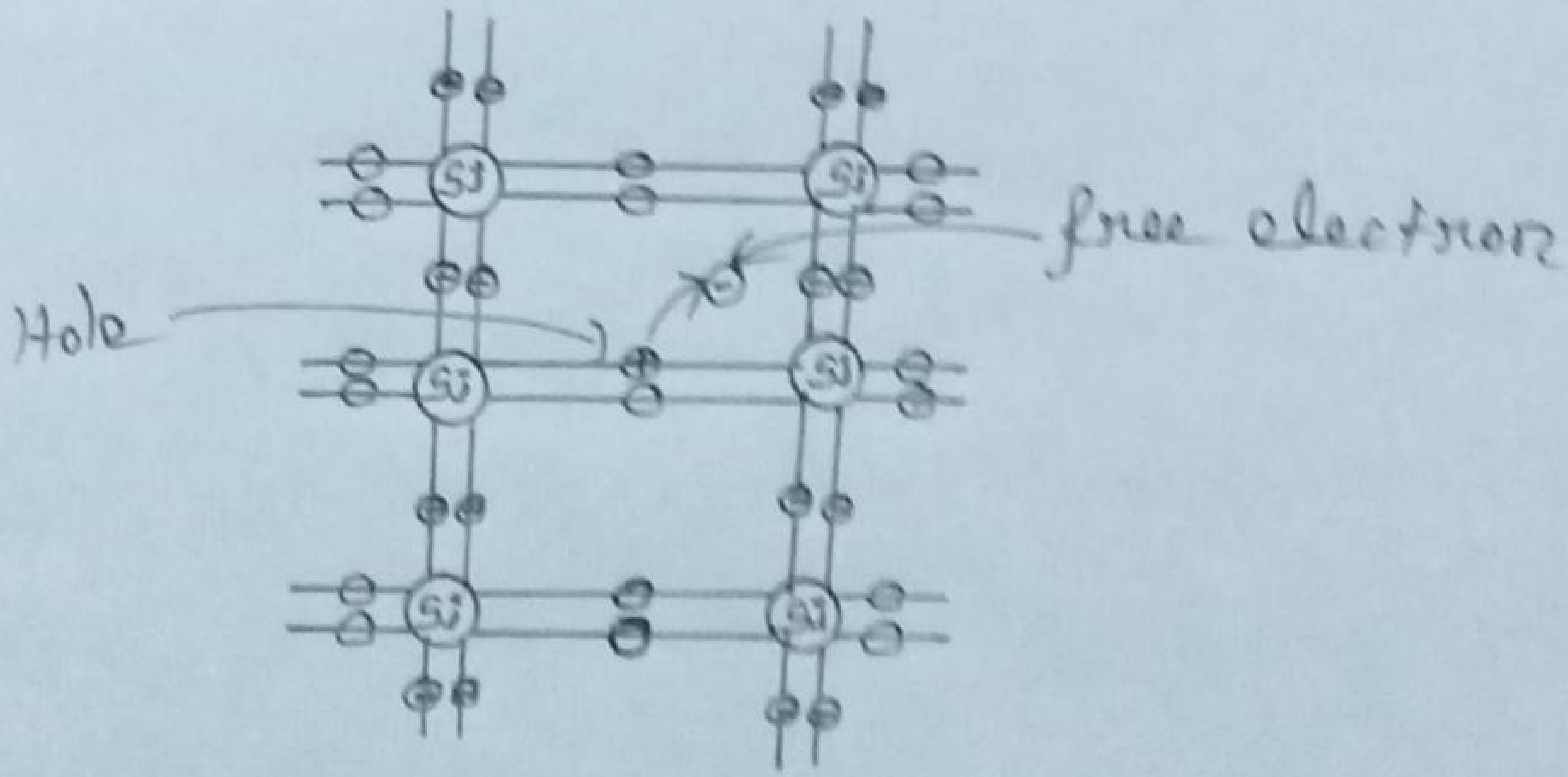

Fig: Bonding structure of Pure Silicon

Each silicon atom shares its four valence electrons with the four neighboring atoms, forming four covalent bonds. In the absence of thermal energy all electrons are bonded to the corresponding atoms and the semiconductor is not conductive.

When the temperature increases, some of the electrons obtain sufficient energy to break up the covalent bond and cross the energy gap.

Energy gap, $W_g = W_c - W_v$

where, W_c - conduction band energy

W_v - valence band energy

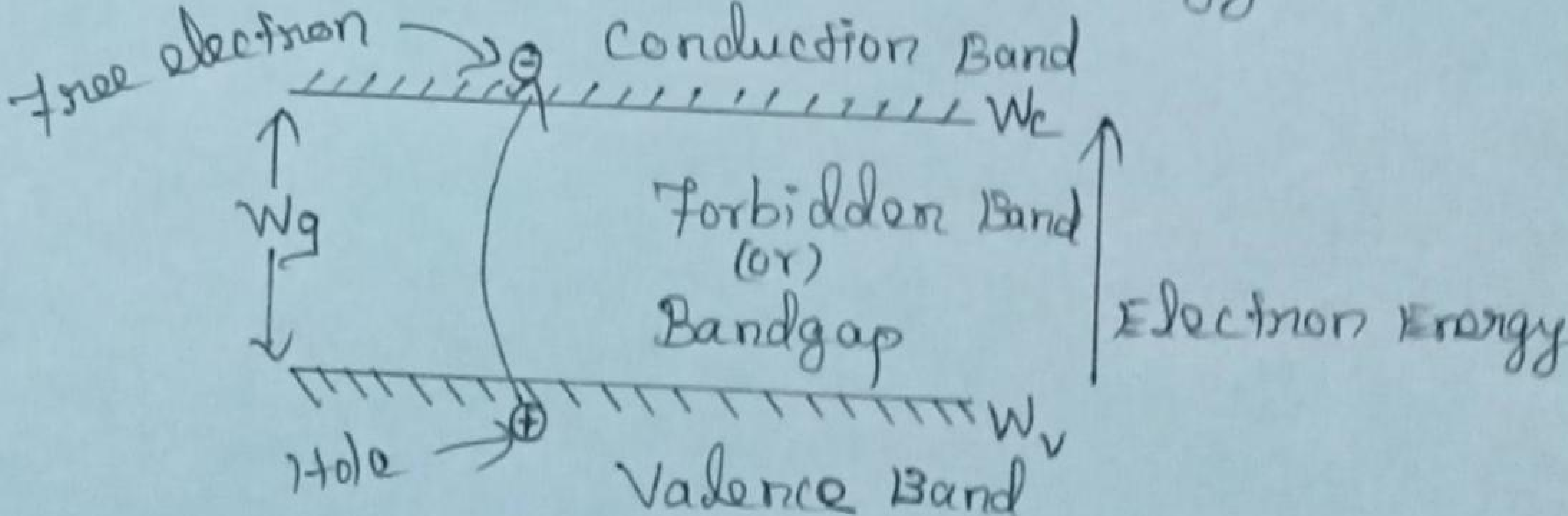

Fig: Energy Band Levels

These electrons form negative charge carriers that allow electric current conduction. When an electron breaks the covalent bond it leaves behind a positively charged vacancy, which can be occupied by another free electron. These types of vacancies are called holes.

Electrons and holes undergo random motion through the semiconductor lattice as a result of the presense of thermal energy. If an electron happens to meet a hole, they recombine and both charge carriers disappear. In thermal equilibrium - equal number of recombinations and generation of holes and electrons.

Bipolar Junction Transistors

Bipolar Junction Transistor is a type of transistor that uses both electrons and holes as charge carriers. Its operation requires that the negatively charged electrons and positively charged holes, so it is named as Bipolar Junction Transistor. BJT is an solid state device in which the current flow between two terminals [collector and emitter] is controlled by the amount of current flow through a third terminal (base). Therefore it is also named as current controlled device.

Construction

The BJT is one of the most widely used active RF elements due to its low cost construction, high operating frequency, low-noise performance and high power handling capacity.

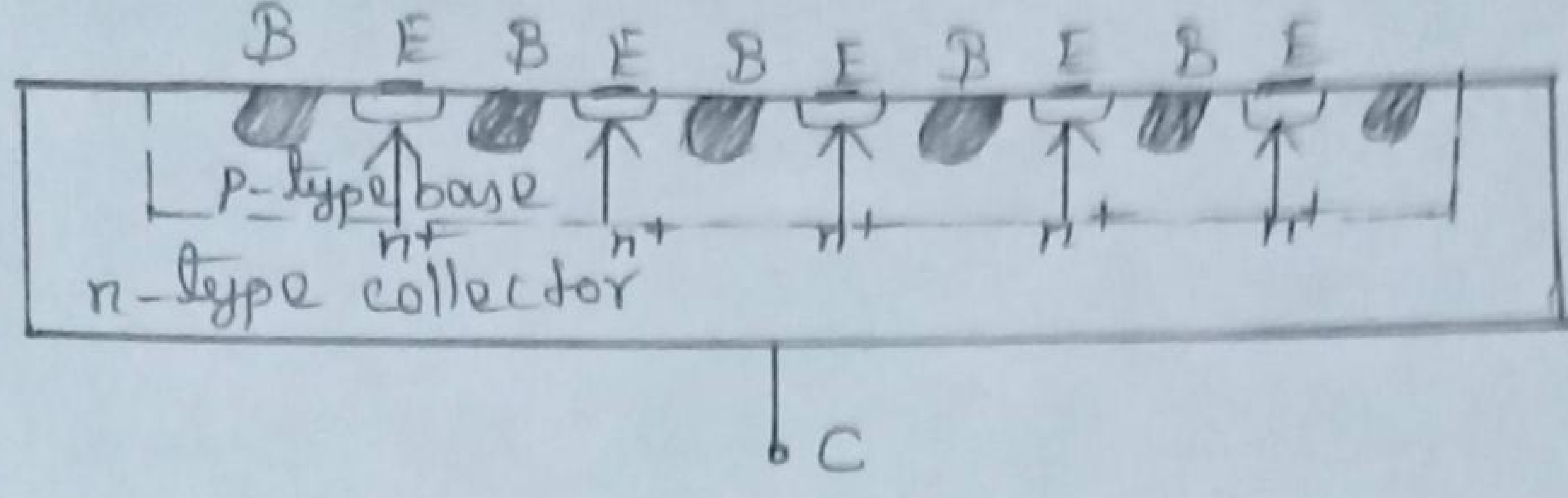

Fig: Interdigitated Structure of High Frequency BJT

The high power capacity is achieved through a special inter-digital emitter-base construction as part of a planar structure. Because of the interleaved construction, the base-emitter resistance is kept at a minimum while not compromising the gain performance. Also, a low base resistance directly improves the signal to noise ratio by reducing the current density through the base-emitter junction and by reducing the random thermal motion in the base.

For frequency applications exceeding 1 GHz it is important to reduce the emitter width to typically less than 1 μm size while increasing the doping to levels of $10^{20} \ldots 10^{21} cm^{-3}$, to both reduce base resistance and increase current gain.

Functionality

In general, two types of BJTs are npn and pnp transistors. The difference between these two types lies in the doping of the semiconductor used to produce base, emitter and collector.

For an npn transistor, collector and emitter are made of n-type semiconductor, while the base is of p-type. For a pnp transistor, collector and emitter are made of p-type semiconductor, while the base is n-type semiconductor. The emitter has the highest and the base has the lowest concentration of doping atoms.

Forward Active Mode

Under forward active mode of operation, the emitter-base diode is operated in forward direction with $V_{BE} = 0.7\,V$ and the base-collector diode is operated in reverse direction. Thus the emitter injects electrons into the base and conversely from the base a hole current reaches the emitter. If maintain the collector-emitter voltage to be larger than the saturation emitter voltage

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voltage and since the base is very thin and lightly doped p-type layer, only a small amount of electrons recombine with the holes supplied through the base current. The majority of electrons reach the base-collector junction and are collected by the applied reverse voltage V_{BC}.

ii) Reverse Active Mode

For the reverse active mode, the collector-emitter voltage is negative and the base-collector diode is forward biased, while the base-emitter diode is operated in reverse direction. Unlike the forward active mode, it is now the electron flow from the collector that bridges the base and reaches the emitter.

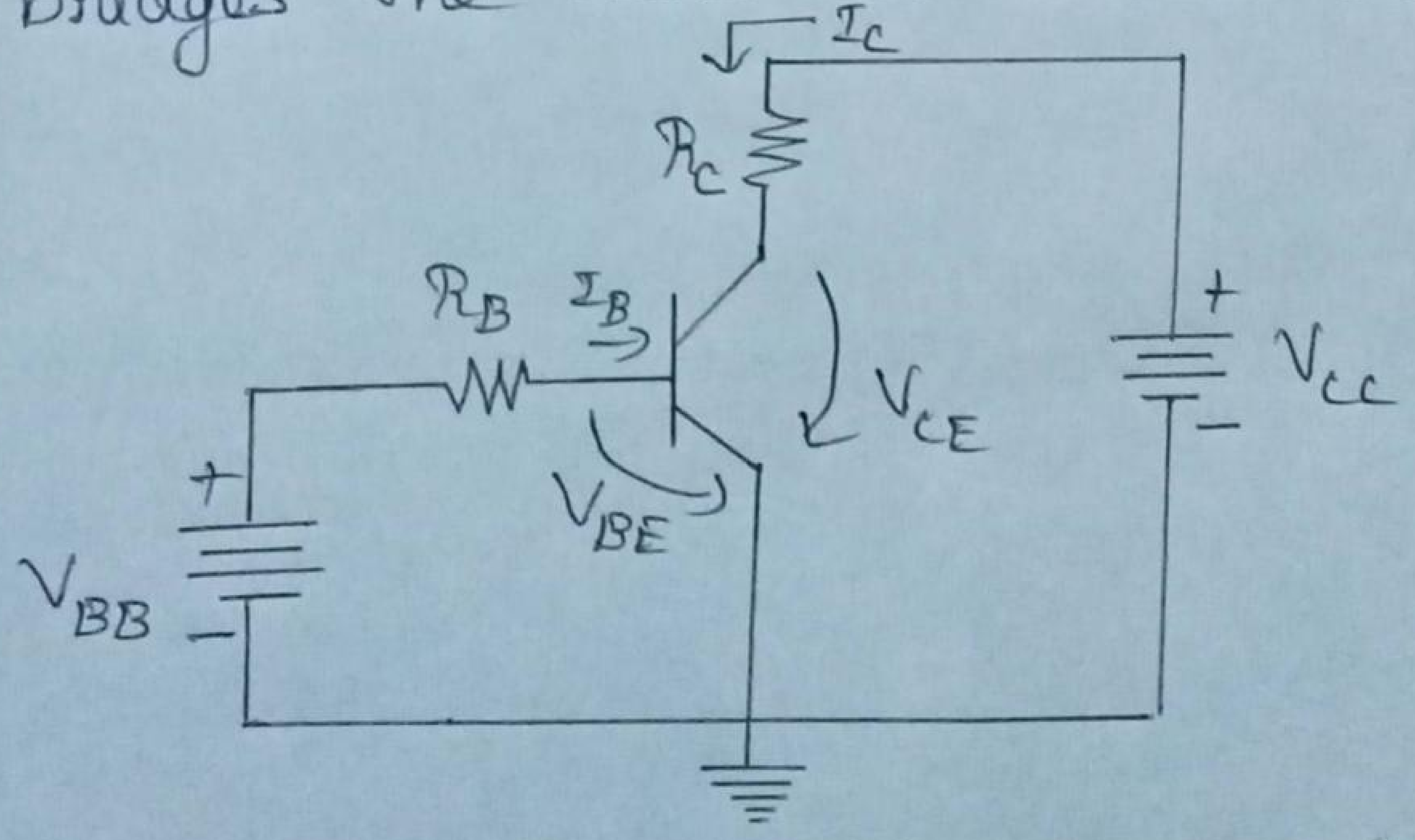

Fig: Biasing circuit for npn BJT in CE configuration

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Saturation Mode:

The saturation mode involves the forward biasing of both the base-emitter and base-collector junctions. This mode of operation plays an important role in switching circuits.

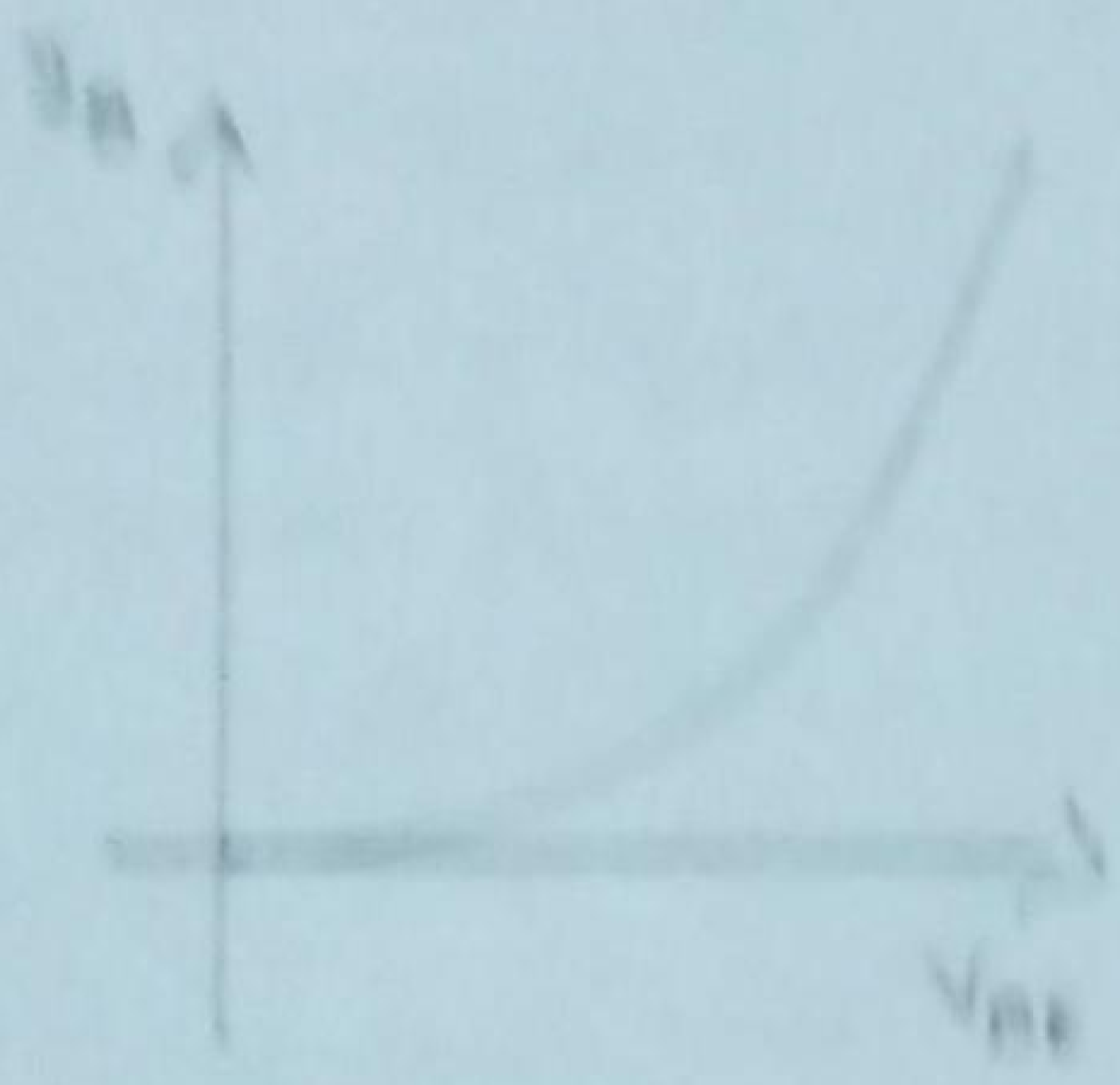

Fig: Input Characteristics

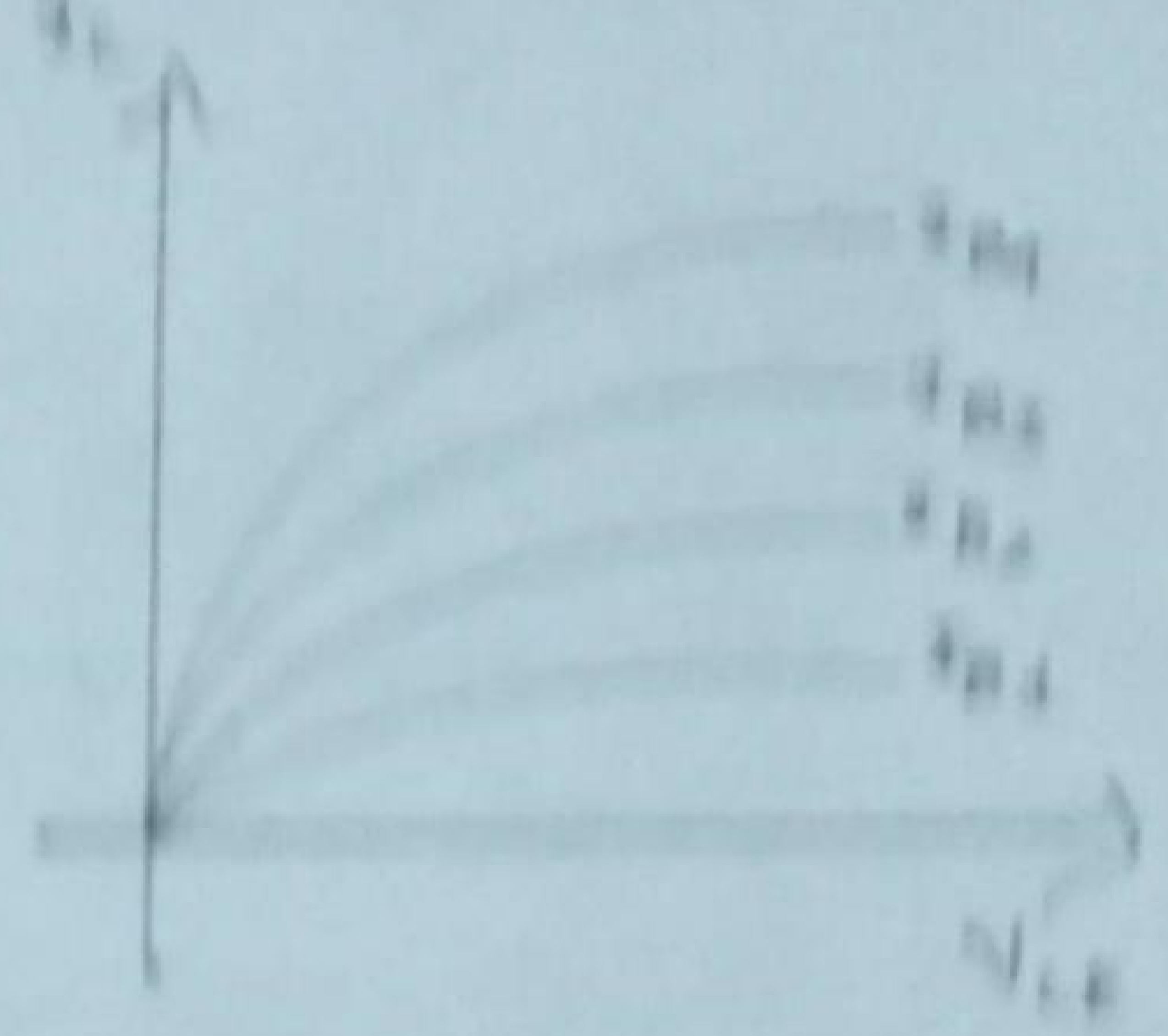

Fig: Output Characteristics

RF Field Effect Transistors

Field effect transistor is a three terminal device namely source, drain and gate. FET uses an electric field to control the flow of current. In field effect transistor, the voltage on one terminal (gate) creates a field that allows or disallows conduction between the other two terminals [source and drain].

Field effect transistors are unipolar devices (or monopolar devices), meaning that only one carrier type, either holes or electrons, contributes to the current flow through the channel. If hole contri-butions are involved for current flow in the FET, then it is named as P-channel field effect transistor. If an electron contributions are involved for current flow in the device, then it is named as N-channel field effect transistor.

FET is a voltage controlled device. A variable electric field controls the current flow from source to drain by changing the applied voltage on the gate (terminal) electrode.

FETs are voltage sensitive devices with high input impedance. Since this input impedance is considerably higher than that of BJTs, FETs are preferred over BJTs for use as the input stage in a multistage amplifier.

Types:

FETs are classified according to how the gate terminal is connected to the conducting channel.

1. Metal Insulator Semiconductor FET (MISFET)

Here the gate terminal is separated from the channel through an insulation layer.

2. Junction FET (JFET)

This type of FET relies on a reverse biased pn junction that isolates the gate from the channel.

3. Metal Semiconductor FET (MESFET)

If the reverse biased pn junction is replaced by a Schottky contact, the channel can be controlled just as in the JFET case.

4. Hetero FET

As the name implies the hetero structures utilize abrupt transitions between layers of different semiconductor materials.

Ex: * GaAlAs to GaAs interface
* GaInAs to GaAlAs interface

Functionality

The depletion mode MESFET may also be operated in an enhancement mode.

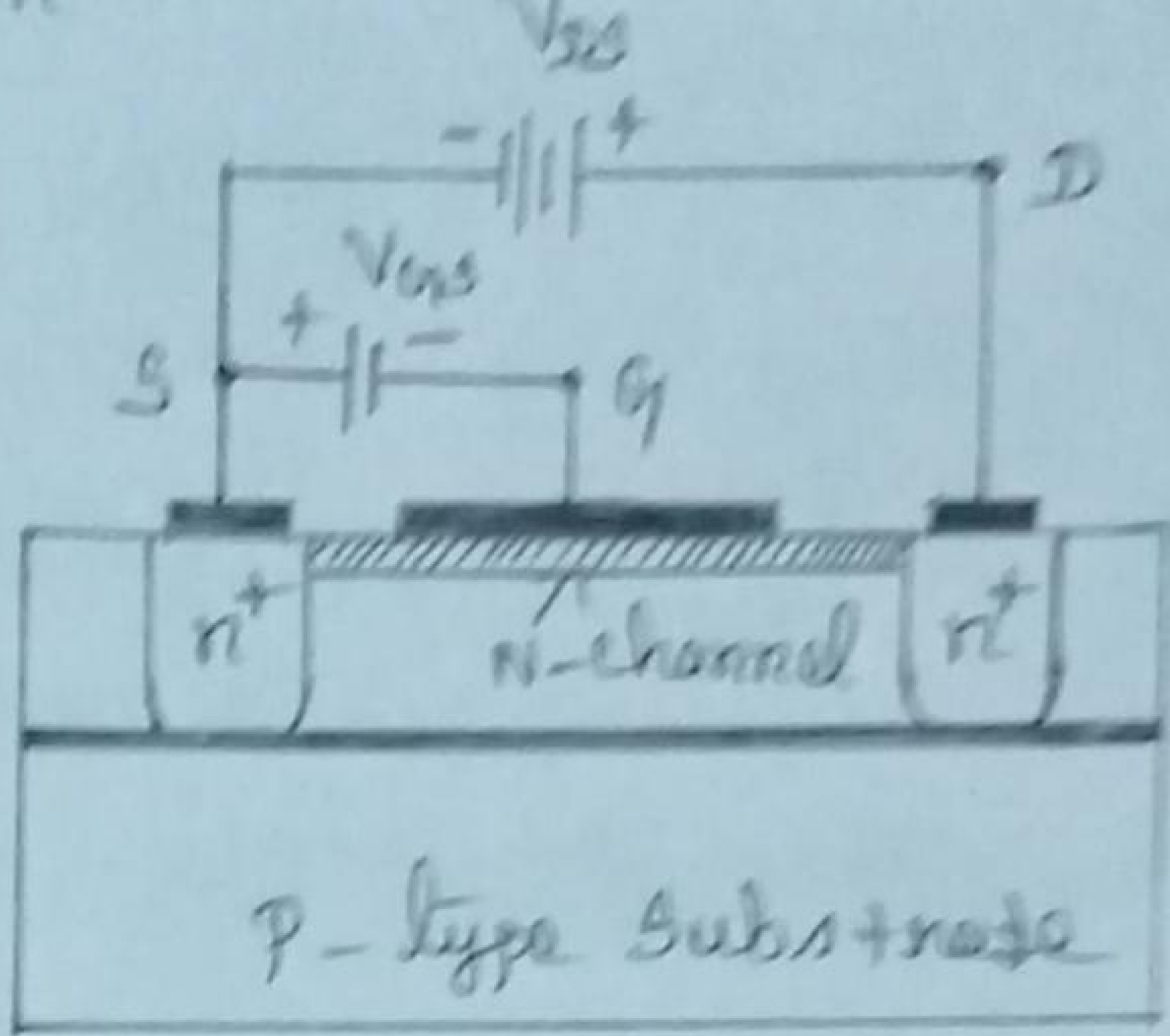

Fig: Functionality of MESFET for different Drain-Source Voltages [Depletion Mode]

With $V_{GS}=0$ and the drain at a positive potential with respect to the source, the electrons flow through the N-channel from source to drain. Therefore the conventional drain current I_D flows through the channel drain to source.

With $V_{GS} < 0$ ie, if the gate voltage is negative, positive charge consisting of holes is induced in the channel through the gate (SiO_2) channel capacitor.

The introduction of the positive charge

causes depletion of mobile electrons in the channel. Thus a depletion region is produced in the channel.

When V_{DS} is increased, I_D increases and it becomes practically constant at a certain value of V_{DS} called the pinch-off voltage. The drain current almost saturated beyond the pinch-off voltage.

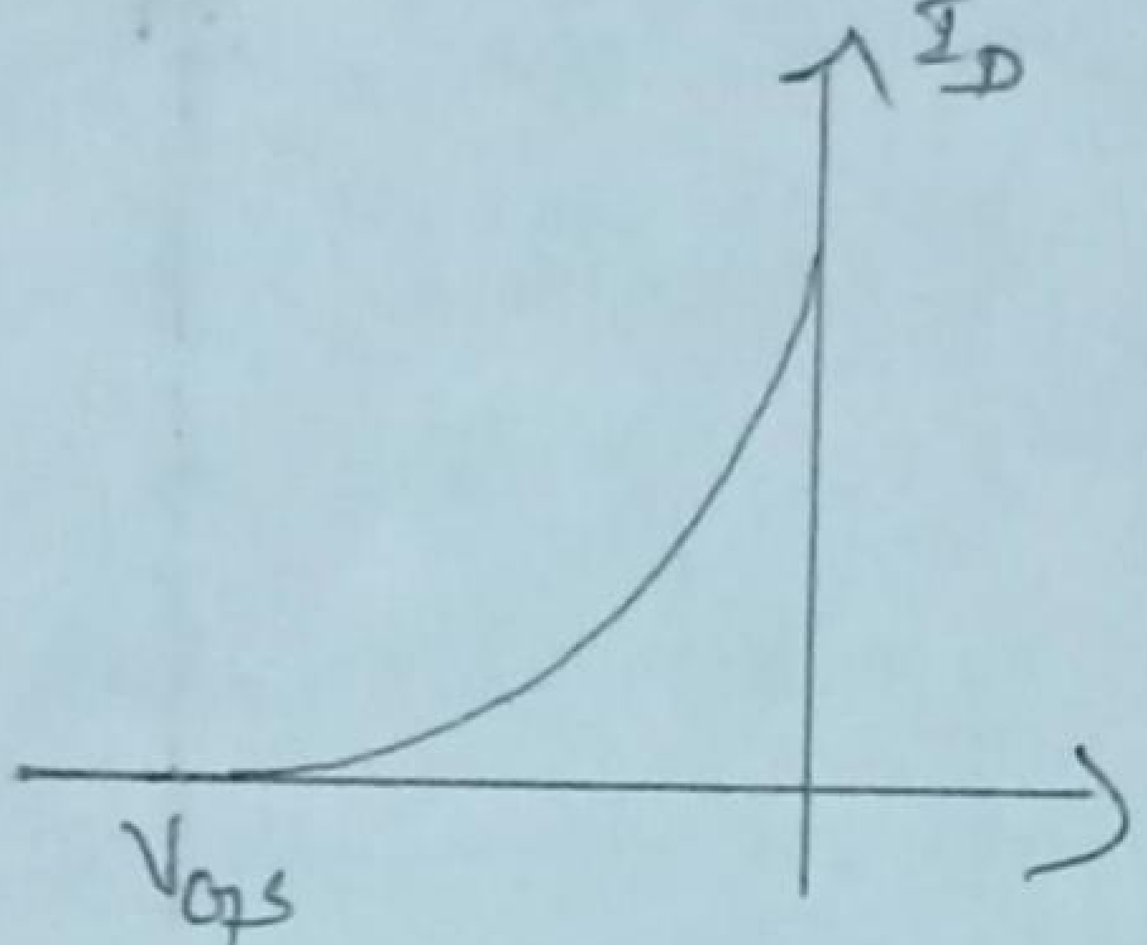

Fig: Transfer characteristics

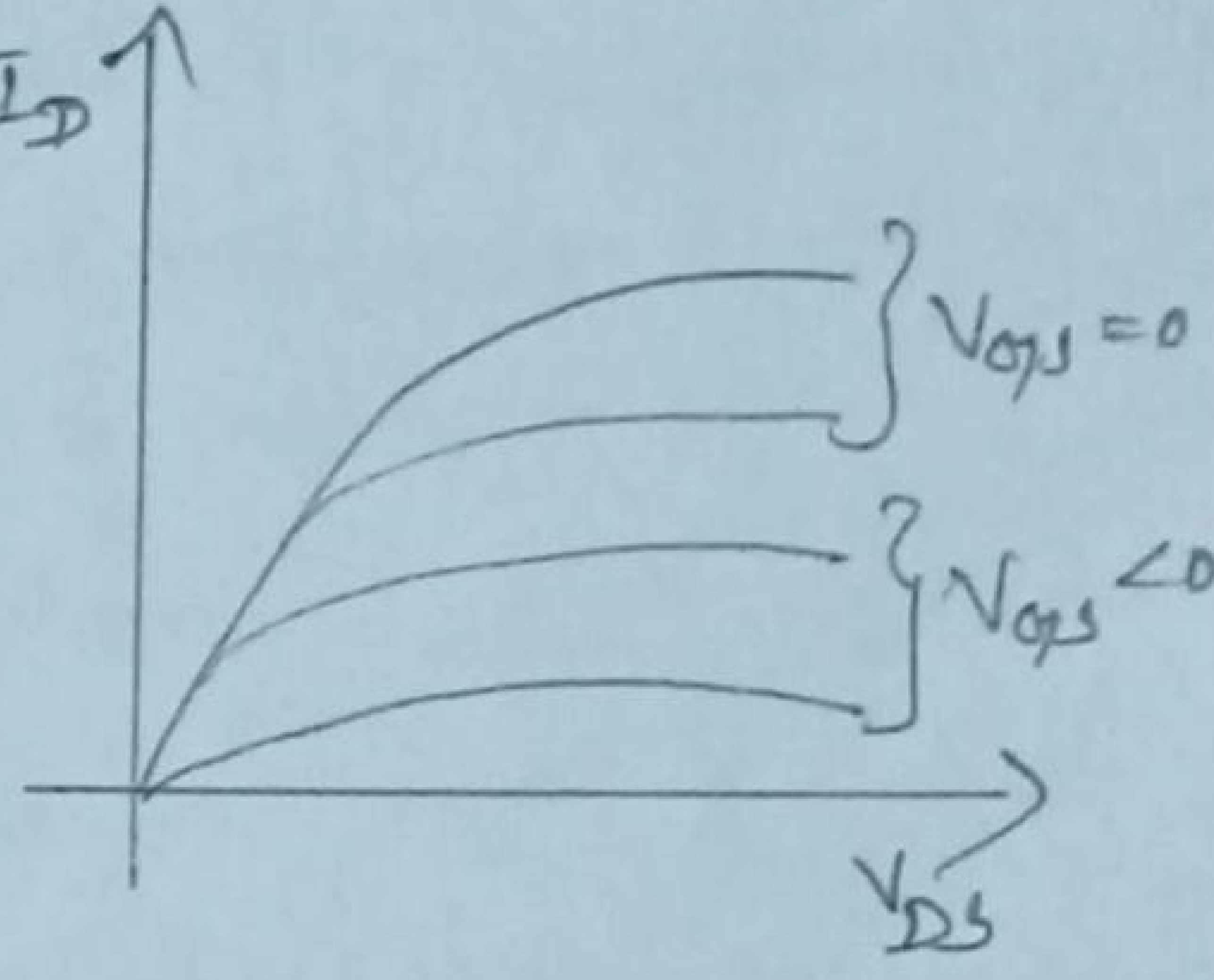

Fig: Output Characteristics

High Electron Mobility Transistors

High electron mobility transistor is a field effect transistor incorporating a junction between two material with different band gaps (ie, heterojunction) as the channel instead of a doped region. Commonly used material combination is GaAs with AlGaAs.

The high electron mobility transistor (HEMT) also known as modulation doped field effect transistor (MODFET), exploits the differences in band gap energy between dissimilar semiconductor materials such as GaAlAs and GaAs in an effort to substantially surpass the upper frequency limit of the MESFET while maintaining low noise performance and high power rating. At present transit frequencies of 100 GHz and above have been achieved.

The high frequency behavior is due to a separation of the carrier electrons from their donor sites at the interface between the doped GaAlAs and undoped GaAs layer (quantum well), where they are confined to a very narrow layer (about 10nm thick) in which motion is possible only parallel to the interface.

Construction

In HEMT, GaAlAs n-doped semiconductor is followed by an undoped GaAlAs spacer layer of the same material, an undoped GaAs layer

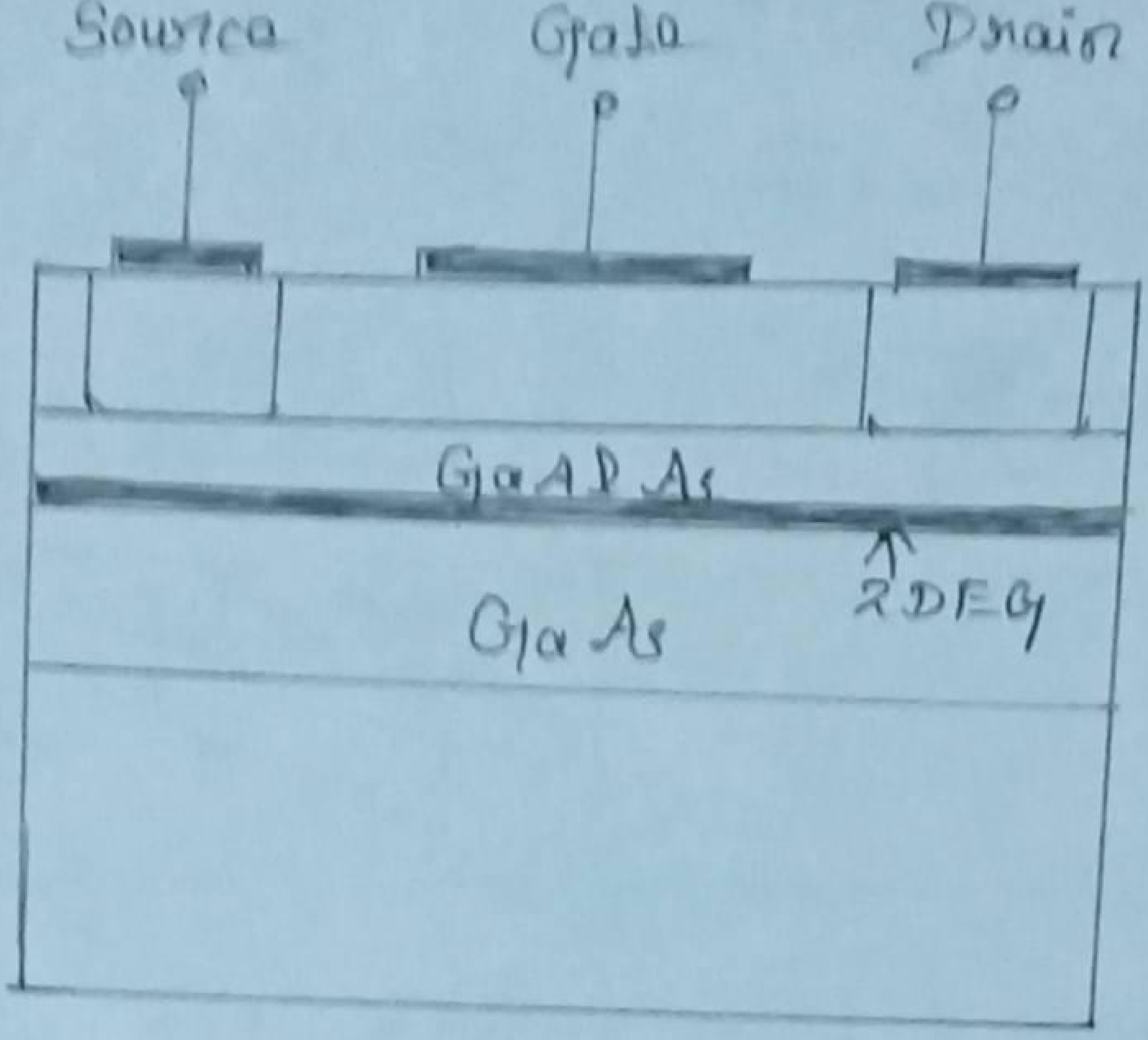

Fig: Heterostructure of Depletion mode HEMT

and a high resistive semi-insulating GaAl substrate.

The 2DEG is formed in the undoped GaAs layer for zero gate bias condition because the Fermi level is above the conduction band so that electrons accumulate in this narrow potential well. The electron concentration can be depleted by applying an increasingly negative gate voltage.

Functionality

The main issue that determines the drain current flow in a HEMT is the narrow interface between the GaAlAs and the GaAs layers. For simplicity, neglect the spacer layer and concentrate at the energy band model.

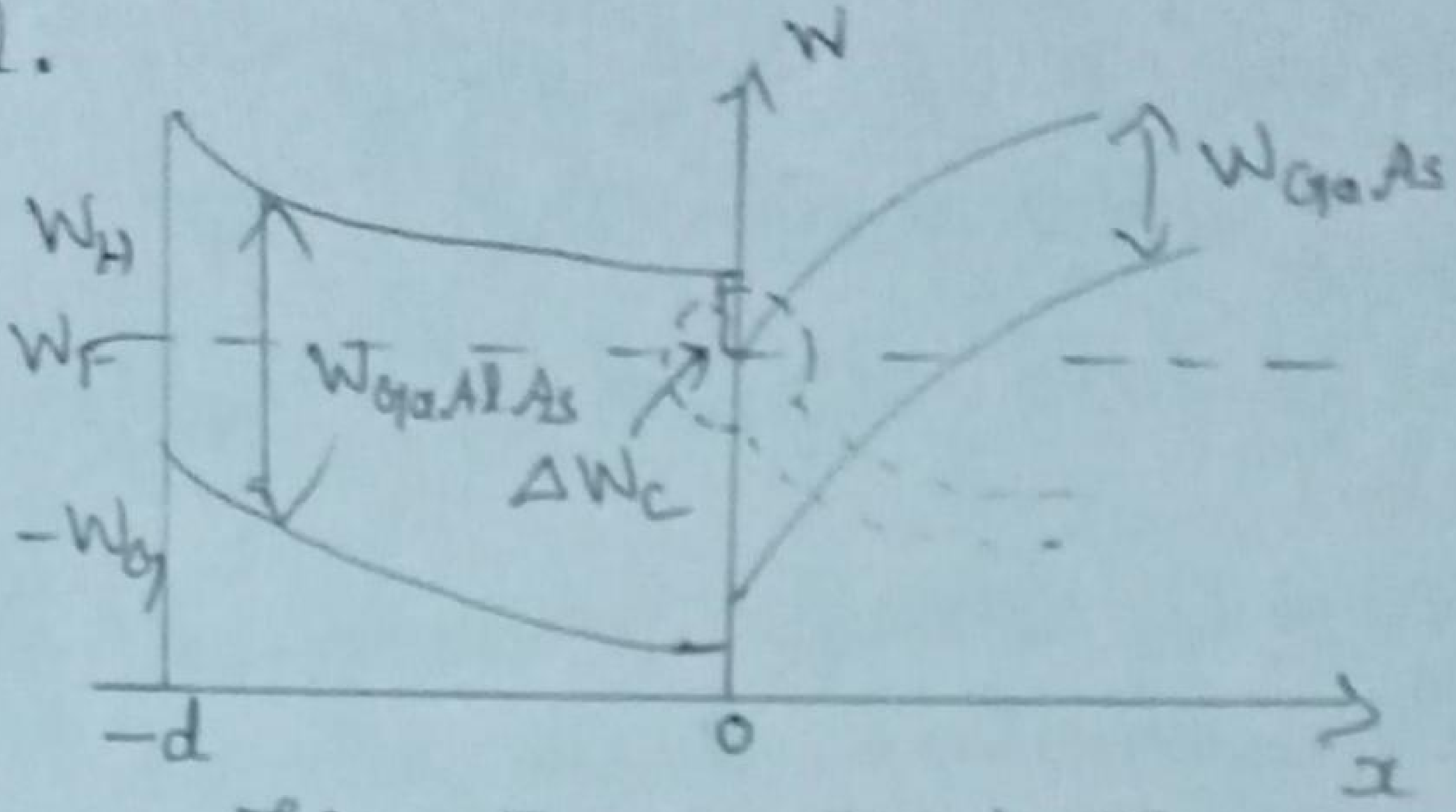

Fig: Energy Band Diagram

A mathematical model is developed by writing the one-dimensional Poisson equation in the form,

$$\frac{d^2V}{dx^2} = \frac{qN_D}{\varepsilon_H} \rightarrow (1)$$

where,

N_D - Donor Concentration

ε_H - Dielectric constant in the GaAlAs heterostructure

The boundary conditions for the potential are imposed such that $V(x=0)=0$ and at the metal semiconductor side $V(x=-d) = -V_b + V_G + \frac{\Delta W_c}{q}$

where,

V_b - barrier voltage

ΔW_c - energy difference in the conduction levels between the n-doped GaAlAs and GaAs

V_G - Gate-source voltage + channel voltage drop

where, $V_G = -V_{GS} + V(y) \rightarrow$ channel voltage drop

To find the potential equation (1) is integrated twice. At the metal-semiconductor we get,

$$V(-d) = \frac{qN_D}{2\varepsilon_H} x^2 - E_y(0)d \longrightarrow (2)$$

which yields,

$$E(0) = \frac{1}{d}\left(V_{GS} - V(y) - V_{TO}\right) \longrightarrow (3)$$

HEMT's threshold voltage V_{TO} is,

$$V_{TO} = V_b - \Delta W_c/q - V_p \longrightarrow (4)$$

wkt,

Pinch-off voltage, $V_p = qN_D d / 2\varepsilon_H \longrightarrow (5)$

From the known electric field at the interface, the electron drain current,

$$I_D = \sigma E_y A = -q\mu_n N_D E W d = q\mu_n N_D \left(\frac{dV}{dy}\right) W d \longrightarrow (6)$$

Here, the current flow is restricted to a very thin layer so that it is appropriate to carry out the integration over a surface charge density Q_s at $x=0$.

The result is,

$$\sigma = -\mu_n Q/W_1 d = -\mu_n Q_s/d \longrightarrow (7)$$

for the surface charge density with Gauss's law,

$$Q_s = \varepsilon_H E(0) \longrightarrow (8)$$

Apply this condition in equ. (6) we get,

$$\int_0^L I_D\,dy = \mu_n W \int_0^{V_{DS}} Q_s\,dV \longrightarrow (9)$$

Using equation (3), the drain current is,

$$I_D L = \mu_n W \int_0^{V_{DS}} \frac{\varepsilon_H}{d}(V_{GS} - V - V_{T0})\,dv \longrightarrow (10)$$

(or) $$I_D = \mu_n \frac{W \varepsilon_H}{Ld}\left\{V_{DS}(V_{GS} - V_{T0}) - \frac{V_{DS}^2}{2}\right\} \longrightarrow (11)$$

Pinch-off occurs when the drain-source voltage is equal to or less than the difference of gate-source and threshold voltages ie., $V_{DS} \leq (V_{GS} - V_{T0})$. If the equality of this condition is substituted in equ (11), we get,

$$I_D = \mu_n \frac{W \varepsilon_H}{2Ld}(V_{GS} - V_{T0})^2 \longrightarrow (12)$$

The threshold voltage allows to determine if the HEMT is operated as an enhancement or depletion type.

For the depletion type,

$$\left.\begin{aligned} & V_{TO} < 0 \\ & \text{(or)} \left\{ V_b - (\Delta W_c / q) - V_p \right\} < 0 \end{aligned}\right\} \rightarrow (13)$$

Substituting the pinch-off voltage $V_P = q N_D d^2 / 2\epsilon$ and solving for 'd' is,

$$d > \left\{ \frac{2\epsilon_H}{q N_D} \left(V_b - \frac{\Delta W_c}{q} \right) \right\}^{1/2} \rightarrow (14)$$

* If d is less than ($V_{TO} > 0$), HEMT operates in an enhancement mode.

Frequency Response

The high frequency performance of the HEMT is determined by the transit time similar to the MESFET. However, the transit time τ is expressed through the electron mobility μ_n and the electric field E of the drain-source voltage according to,

$$\tau = \frac{L}{v_{sat}} = \frac{L}{\mu_n E_y} = \frac{L^2}{\mu_n V_{DS}} \longrightarrow (15)$$

Transit frequency, $f_T = \frac{1}{2\pi\tau}$

$f_T \approx 190\,GHz$ for the gate length of $1.0\,\mu m$

Mobility of $\mu_n = 8000\ cm^2/V\text{-}s$ at a typical drain voltage V_{DS} of $1.5V$

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BASIC CONCEPTS OF RF DESIGN

Radio Frequency (RF) refers to the rate of oscillation of electromagnetic radio waves in the range of 3KHz to 300GHz. Radio waves are electromagnetic waves propagated by an antenna which is used for communication.

Radio Frequency (RF) engineering is a subset of electronic engineering involving the application of transmission line, waveguide, antenna and electromagnetic field principles to the design and application of devices that produce or utilise signals with in the radio band. The frequency range of about 20KHz up to 300GHz.

RF components are Attenuators, DC blocks, filters, Phase shifters and tappers ... etc ...

RF refers to the use of electromagnetic radiation for transferring information between two circuits that have no direct electrical connection. Time varying voltages and currents generate electro-magnetic energy that propagates in the form of waves.

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MIXERS

RF mixer is a three port passive or active device that can modulate or demodulate a signal. The purpose is to change the frequency of an electromagnetic signal while preserving every other characteristic (phase, amplitude) of the initial signal.

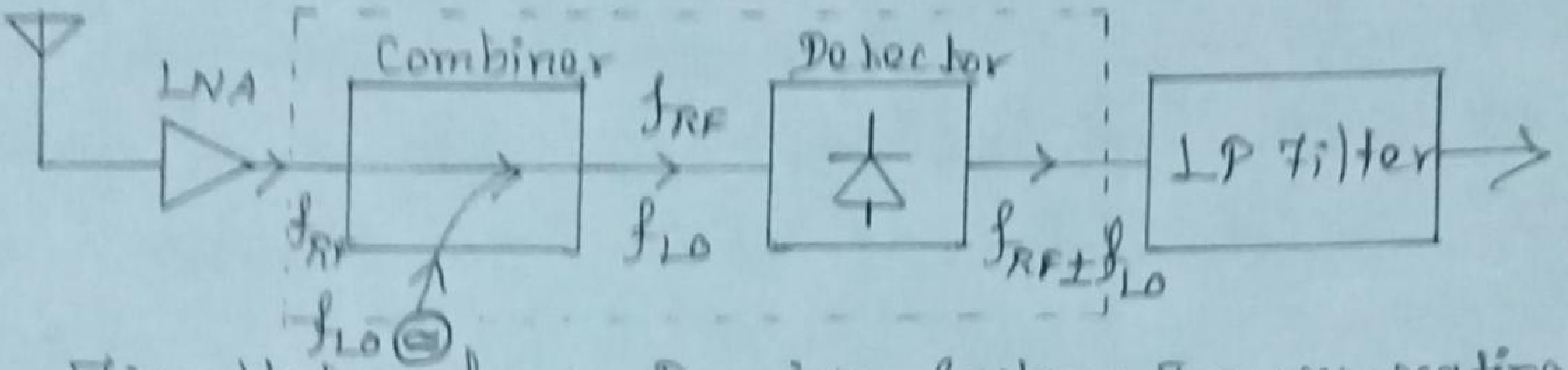

Fig: Heterodyne Receiver System Incorporating a Mixer

Mixers are commonly used to multiply signals of different frequencies in an effort to achieve frequency translation. The motivation for this translation stems from the fact that filtering out a particular RF signal channel centered among many densely populated, narrowly spaced neighboring channels would require extremely high 'Q' filters.

Here the received RF signal is, after preamplification in a low noise amplifier, supplied to a mixer. It's task is to multiply the input signal of centre frequency f_{RF} with a local oscillator (LO) frequency f_{LO}. The signal obtained after the mixer contains the frequencies $f_{RF} \pm f_{LO}$, of which, after low pass filtering, the lower frequency component $f_{RF} - f_{LO}$, known as the intermediate frequency is selected for further processing.

The two key ingredients constituting a mixer are the combiner and detector. The combin-er can be implemented through the use of a 90° (or 180°) directional coupler. The detector employs a single diode as a non linear device.

Low Noise Amplifier

Low noise amplifier (LNA) is an electronic amplifier that amplifies a very low power signal without significantly degrading its signal to noise ratio. Low noise amplifier is commonly found in all receivers. Its role is to boost the received signal a sufficient level above the noise figure so that it can be used for additional processing. The noise figure of the low noise amplifier directly limits the sensitivity of the receiver.

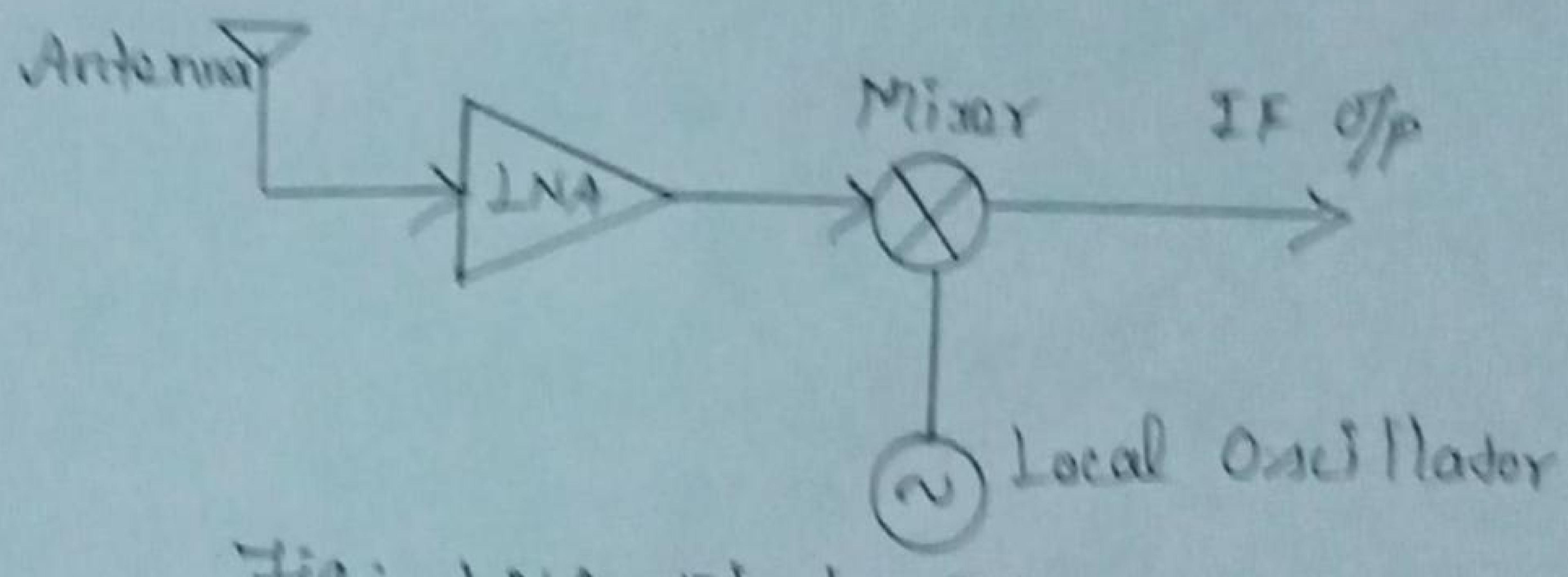

Fig: LNA Block Diagram

Low noise amplifiers are used to amplify signals of very low strength, usually from an antenna where signals are barely recognizable and should be amplified without adding any noise, otherwise important information might be lost.

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Low noise amplifiers are a significant parts of a receiver circuit whereby the received signal is processed and converted into information. They (LNA) are designed to be close to the receiving device so that there is minimum loss due to interference.

As the name suggests, they (LNA) add a minimum amount of noise in the received signal because any more would highly corrupt the already weak signal. When the signal to noise ratio is high and needs to be degraded by around 50% and power needs to be boosted, an LNA is used. An LNA is the first component of a receiver to intercept a signal, making it a vital part in the communication process.

Applications

Low noise Amplifiers are used in,

* Communication receivers such as in,
    * cellular telephones
    * GPS receivers
    * Wireless LANs
  and * Satellite communications

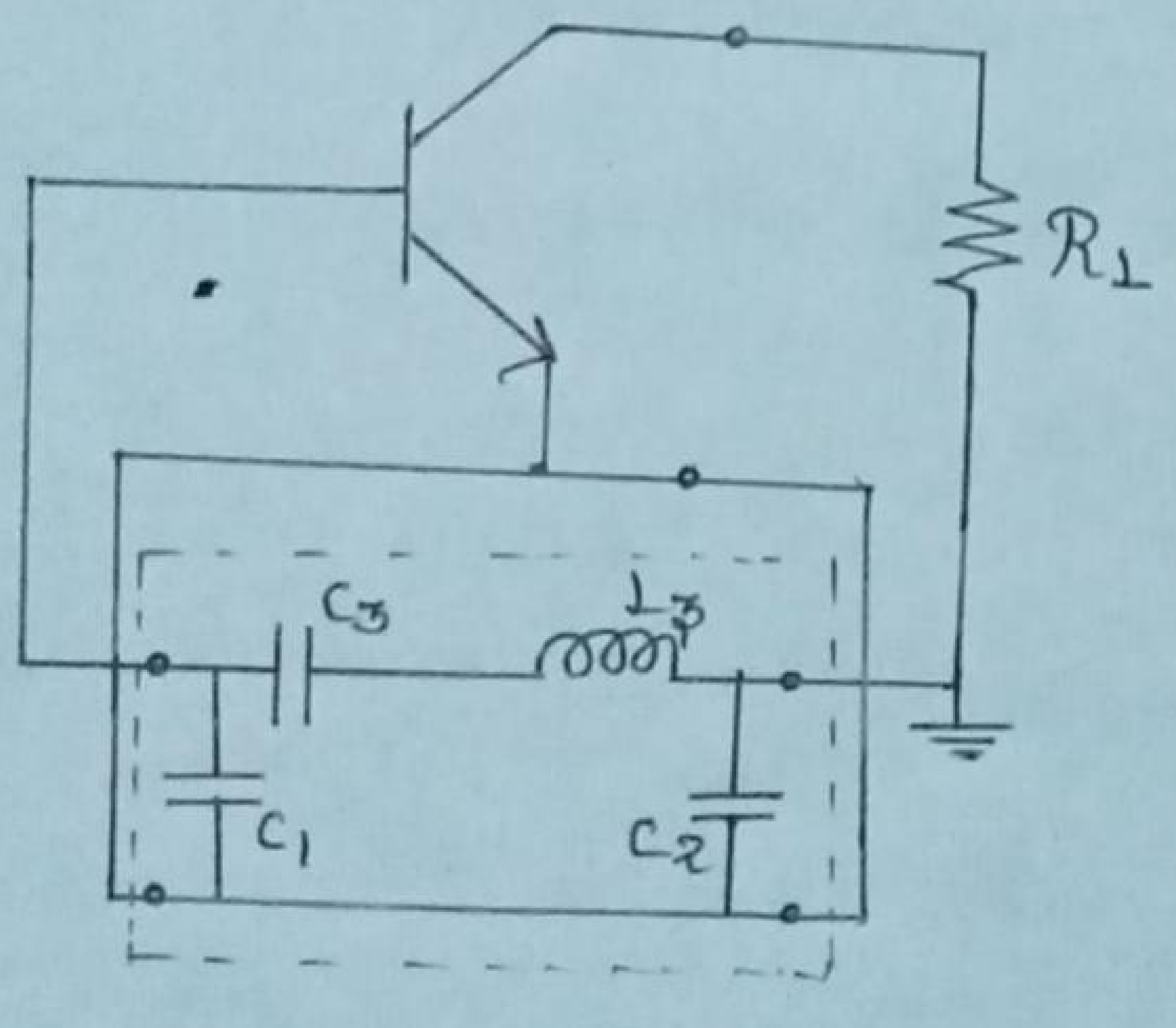
VOLTAGE CONTROLLED OSCILLATORS

Voltage controlled oscillator is an electronic oscillator whose oscillation frequency is controlled by a voltage input. The applied input voltage determines the instantaneous oscillation frequency.

Voltage controlled oscillator is an oscillator whose output frequency is directly related to the voltage at its input.

In voltage controlled oscillator, as the input voltage or control voltage increases, the capacitance gets reduced. Hence, the control voltage and frequency of oscillations are directly proportional.

Commonly used VCO circuits are the Clapp Oscillators and Colpitts oscillators, varactor diode oscillators.



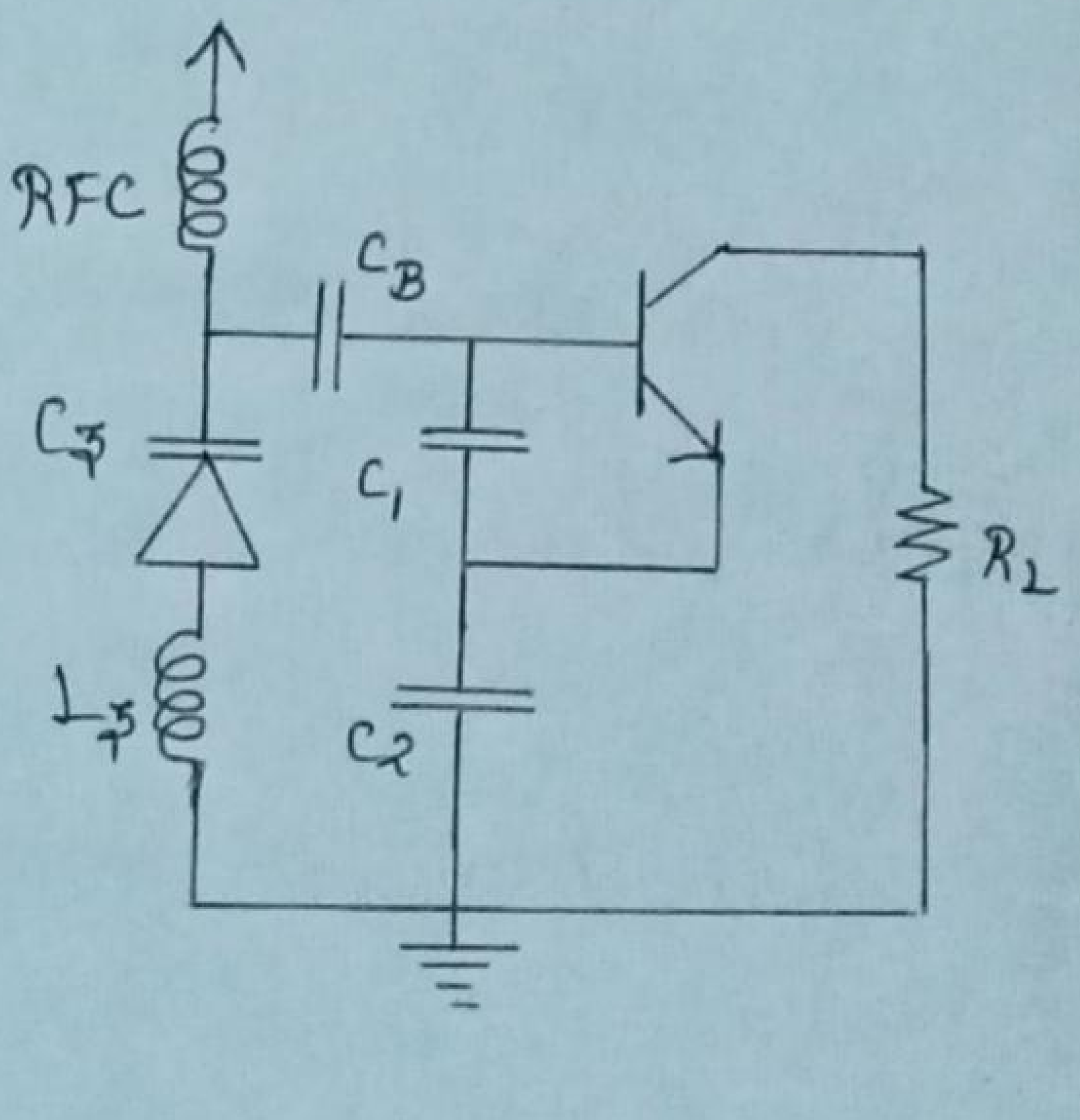


Fig: Varactor Diode Oscillator

The variable capacitance of a varactor diode is,

$$C_V = C_{V0}\left[1 - V_Q/V_{diff}\right]^{-1/2}$$

* C_V is affected by the reverse bias voltage V_Q

→ For analyzing purpose varactor diode oscillator circuit is modified by using simple BJT hybrid model

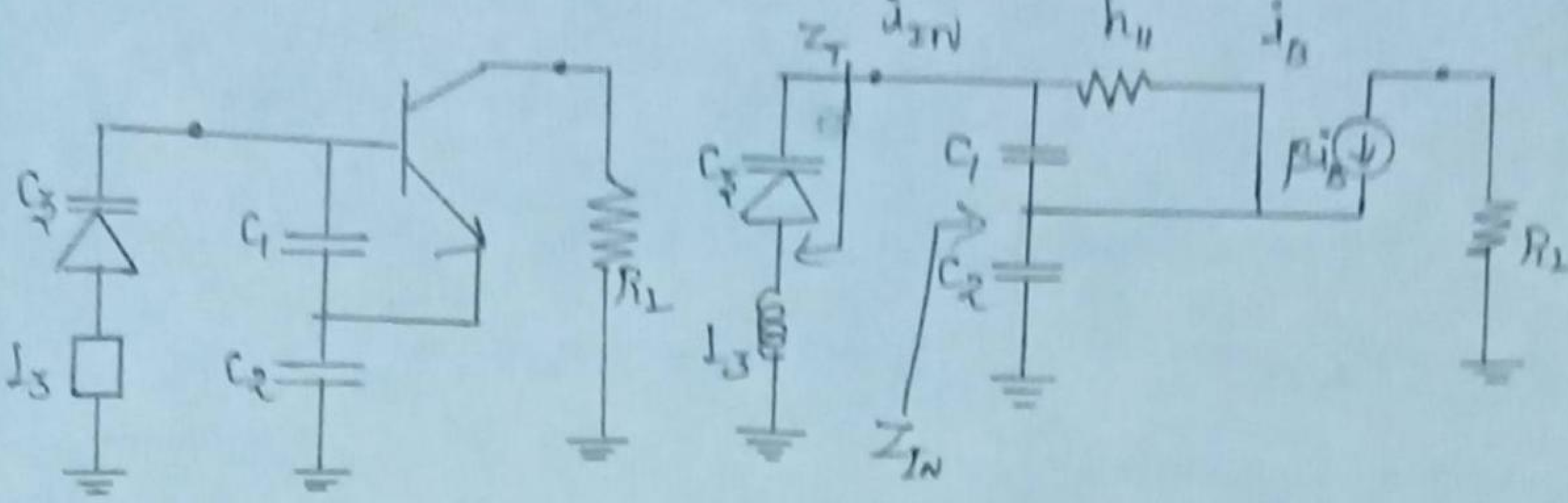


Fig: Circuit Analysis of Varactor Diode Oscillator

* The input impedance is calculated from two equations

$$V_{IN} - i_{IN}X_{C1} - i_{IN}X_{C2} + i_B X_{C1} - \beta i_B X_{C2} = 0 \quad \rightarrow (1)$$

$$h_{11} i_B + i_B X_{C1} - i_{IN} X_{C1} = 0 \quad \rightarrow (2)$$

Rearranging the above two equation leads to,

$$I_{IN} = \frac{1}{h_{11} + X_{C1}}\left\{h_{11}(X_{C1} + X_{C2}) + X_{C1}X_{C2}(1+\beta)\right\} \rightarrow (3)$$

For further simplification,

let $(1+\beta) \approx \beta$, assume $h_{11} >> X_{C1}$

now equ. (3) becomes,

$$I_{IN} = \frac{1}{j\omega}\left[\frac{1}{C_1} + \frac{1}{C_2}\right] - \frac{\beta}{h_{11}}\left(\frac{1}{\omega^2 C_1 C_2}\right) \rightarrow (4)$$

* but, input resistance is negative

∴ with $g_m = \beta / h_{11}$

$$R_{IN} = -\frac{g_m}{\omega^2 C_1 C_2} \longrightarrow (5)$$

$$X_{IN} = \frac{1}{j\omega C_{IN}} \longrightarrow (6)$$

where,

$$C_{IN} = \frac{C_1 C_2}{C_1 + C_2}$$

The condition for resonance frequency is,

$$X_1 + X_2 + X_3 = 0 \longrightarrow (7)$$

(or) $$j\omega_0 L_3 - \frac{1}{\omega_0 C_3} - \frac{1}{j\omega_0}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = 0 \longrightarrow (8)$$

Resonant frequency, $f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{L_3}\left[\frac{1}{C_3} + \frac{1}{C_2} + \frac{1}{C_1}\right]}$

In order to create sustained oscillations, the combined resistance of the varactor diode must be equal to or less than $|R_{IN}|$.

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POWER AMPLIFIERS

Power amplifier is an electronic amplifier designed to increase the magnitude of power of a given input signal. The power of the input signal is increased to a level high enough to drive loads of output devices like speakers, headphones, RF transmitters... etc...

RF power amplifier designs are differ from low frequency circuit approaches and it requires special considerations. Most of the power amplifiers can oscillate when terminated with certain source and load impedances.

Matching network can help stabilize the amplifier by keeping the source and load impedances in the appropriate range. In an power amplifier design process, the stability analysis is a first step. Gain and Noise figure circles are the basic requirements needed to develop an amplifier circuits to meet the requirements of gain, gain flatness, output power, bandwidth and bias conditions.

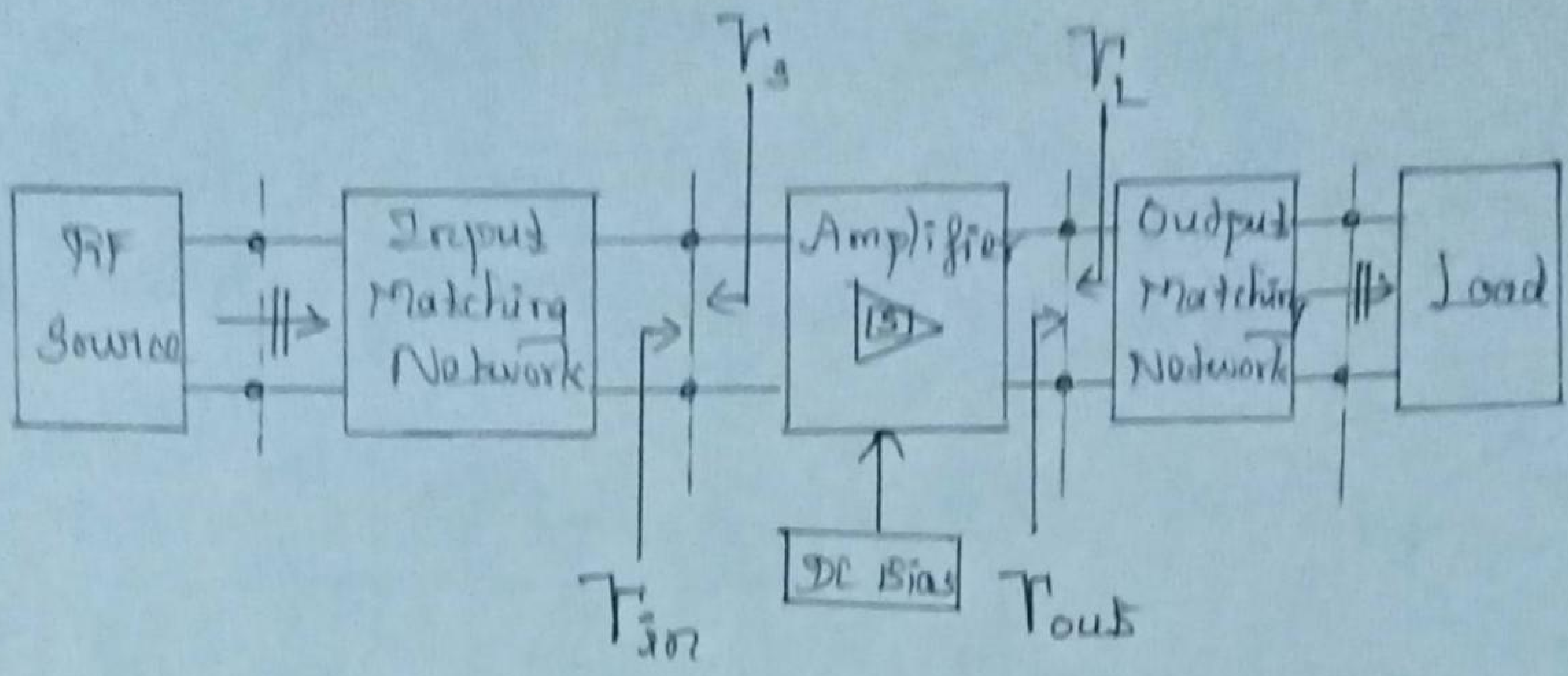

Fig: Amplifier System

Figure shows a generic single stage amplifier configuration with an input and output matching networks. Input and output matching networks are needed to reduce an undesired reflection and also to improve the power flow capabilities. Power amplifier is characterized by its s-parameter matrix at a particular DC bias point.

The key parameters of an amplifier to evaluate its performance are,

* Gain and Gain Flatness
* Operating frequency and bandwidth
* Output power
* Power supply requirements
* Input and Output reflection co-efficients
* Noise Figure

AMPLIFIER POWER RELATIONS

There are various power gain definitions that are critical to the understanding of how an RF

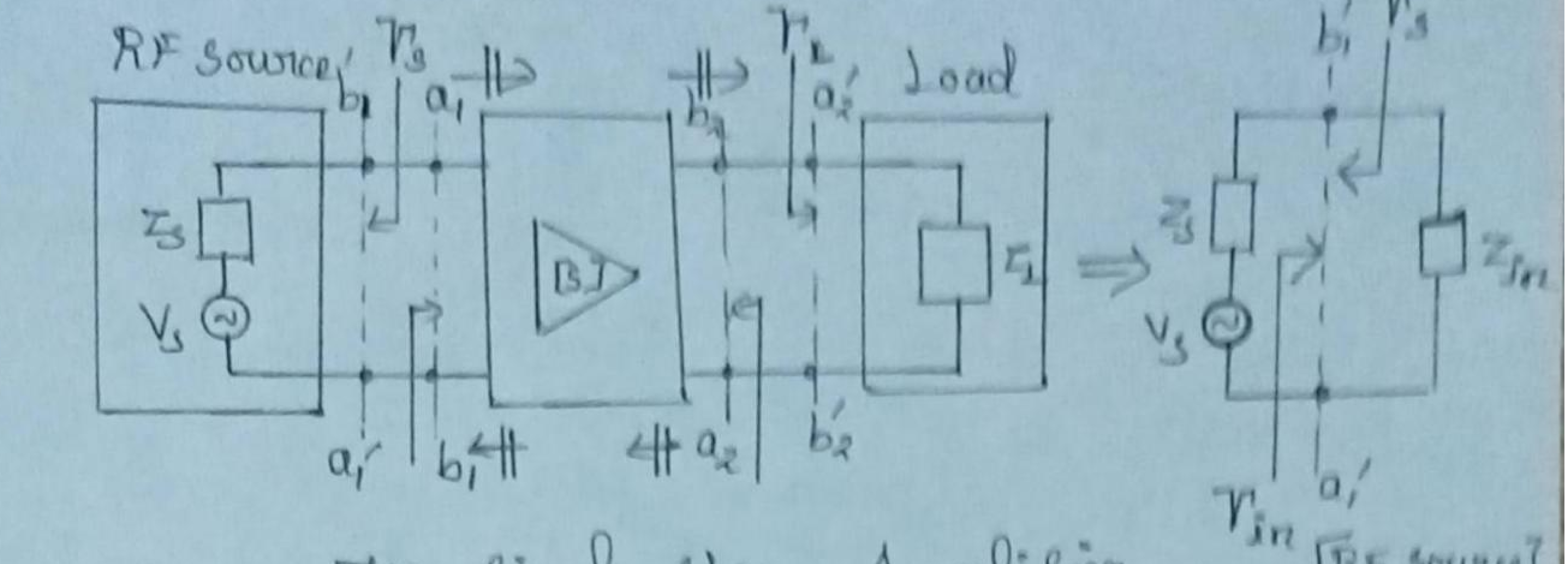

Fig: Single stage Amplifier

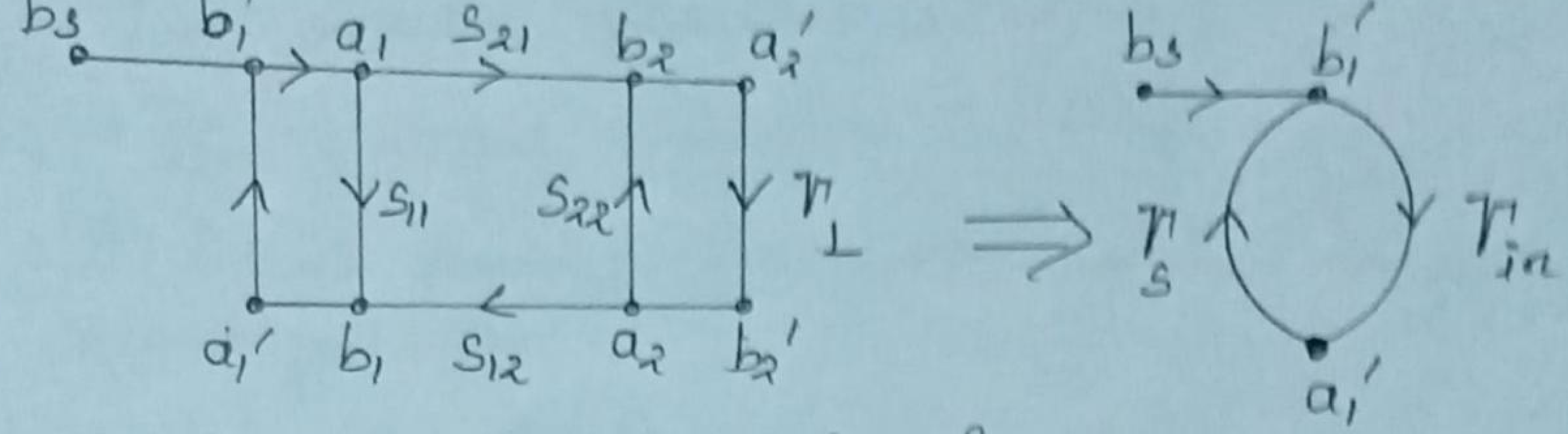

Fig: Signal Flow Graph

power amplifier functions. For this reason, in all the power relations of circuit two matching networks are assumed which includes in the source and load impedances.

RF Source

In the input side, the RF source is connected to the amplifier network.

* Incident Wave Power (P_{inc})

The incident wave power at node b_1' is,

$$P_{inc} = \frac{|b_1'|^2}{2} \longrightarrow (1)$$

$$\Rightarrow P_{inc} = \frac{1}{2} \frac{|b_s|^2}{1-|\Gamma_{in}\Gamma_s|} \longrightarrow (2)$$

where,

Source node, $b_s = \frac{\sqrt{Z_0}}{Z_s + Z_0} V_s \longrightarrow (3)$

Z_0 - characteristic impedance

Z_s - source impedance

Γ_s - source reflection co-efficient

Γ_{in} - Input reflection co-efficient

* Incident power is the power launched towards the amplifier

* Input Power (P_{in})

Actual Input Power observed at the input terminals of the amplifier is composed of the incident and reflected power waves. It is expressed in terms of incident wave reflection co-efficient Γ_{in}.

$$P_{in} = P_{inc}\left[1-|\Gamma_{in}|^2\right] \longrightarrow (4)$$

substitute the value of P_{inc},

$$P_{in} = \frac{|b_s|^2}{|1-\Gamma_{in}\Gamma_s|^2}\left[1-|\Gamma_{in}|^2\right] \longrightarrow (5)$$

* Maximum Power Transfer

If the input impedance is matched with complex conjugate of source impedance $[Z_{in} = Z_s^*]$ or interms of reflection co-efficient $[\Gamma_{in} = \Gamma_s^*]$, then maximum power will be transferred from the source to the amplifier.

Amplifier Power, $P_A = P_{in}\Big|_{\Gamma_{in} = \Gamma_s^*}$

$$= \frac{1}{2}\frac{|b_s|^2}{|1-\Gamma_{in}\Gamma_s|^2}\left[1-|\Gamma_{in}|^2\right]\Bigg|_{\Gamma_{in}=\Gamma_s^*}$$

$$= \frac{1}{2}\frac{|b_s|^2}{|1-\Gamma_s^*\Gamma_s|^2}\left[1-|\Gamma_s|^2\right]$$

$$= \frac{1}{2}\frac{|b_s|^2}{|1-|\Gamma_s|^2|^{\cancel{2}}}\left[\cancel{1-|\Gamma_s|^2}\right]$$

$$P_A = \frac{1}{2}\frac{|b_s|^2}{\left[1-|\Gamma_s|^2\right]} \rightarrow (6)$$

If $\Gamma_{in} = 0$ and $\Gamma_s \neq 0$ then equ.(2) becomes,

$$P_{inc} = \frac{|b_s|^2}{2}$$

Transducer Power Gain(G_T)

Transducer power gain is the gain of the amplifier when placed between the source and load.

$$G_T = \frac{\text{Power delivered to the Load}}{\text{Available power from the source}}$$

i.e., $G_T = \frac{P_L}{P_A}$

$$P_L = \frac{1}{2}|b_2|^2 \cdot (1-|\Gamma_L|^2) \longrightarrow (1)$$

$$P_A = \frac{1}{2}\frac{|b_s|^2}{(1-|\Gamma_s|^2)} \longrightarrow (2)$$

now, $$G_T = \frac{\cancel{\frac{1}{2}}|b_2|^2 \cdot (1-|\Gamma_L|^2)}{\cancel{\frac{1}{2}}\frac{|b_s|^2}{(1-|\Gamma_s|^2)}}$$

$$G_T = \frac{|b_2|^2}{|b_s|^2}(1-|\Gamma_L|^2)(1-|\Gamma_s|^2) \longrightarrow (3)$$

From the signal flow graph,

$$b_2 = \frac{S_{21}a_1}{1-S_{22}\Gamma_L} \longrightarrow (4)$$

$$b_s = \left[1-\left[S_{11}+\frac{S_{21}S_{12}\Gamma_L}{1-S_{22}\Gamma_L}\right]\Gamma_s\right]a_1$$

$$= \left\{ \left[1 - S_{11} - \frac{S_{21} S_{12} \Gamma_1}{1 - S_{22} \Gamma_1} \right] \Gamma_s \right\} a_1$$

$$= \left\{ \left[(1 - S_{11}) - \frac{S_{21} S_{12} \Gamma_1}{(1 - S_{22} \Gamma_1)} \right] \Gamma_s \right\} a_1$$

$$b_s = \left[\frac{(1 - S_{11} \Gamma_s)(1 - S_{22} \Gamma_1) - S_{21} S_{12} \Gamma_1 \Gamma_s}{1 - S_{22} \Gamma_1} \right] a_1 \rightarrow (5)$$

The ratio $\frac{b_2}{b_s}$ is,

$$\frac{b_2}{b_s} = \frac{S_{21} \cancel{a_1} / \cancel{(1 - S_{22} \Gamma_1)}}{\frac{\left[(1 - S_{11} \Gamma_s)(1 - S_{22} \Gamma_1) - S_{21} S_{12} \Gamma_1 \Gamma_s \right] \cancel{a_1}}{\cancel{(1 - S_{22} \Gamma_1)}}}$$

$$\frac{b_2}{b_s} = \frac{S_{21}}{(1 - S_{11} \Gamma_s)(1 - S_{22} \Gamma_1) - S_{21} S_{12} \Gamma_1 \Gamma_s} \rightarrow (6)$$

Substitute equ.(6) in equ.(3) we get,

$$G_T = \frac{S_{21}^2 (1 - |\Gamma_L|^2)(1 - |\Gamma_s|^2)}{\left| \left[(1 - S_{11} \Gamma_s)(1 - S_{22} \Gamma_1) - S_{21} S_{12} \Gamma_1 \Gamma_s \right] \right|^2} \rightarrow (7)$$

In terms of input reflection co-efficient equ.(7) becomes

$$G_T = \frac{|S_{21}|^2 (1 - |\Gamma_L|^2)(1 - |\Gamma_s|^2)}{|1 - \Gamma_s \Gamma_{in}|^2 |1 - S_{22} \Gamma_L|^2} \rightarrow (8)$$

In terms of output reflection co-efficient equ.(7) becomes,

$$G_T = \frac{|S_{21}|^2 (1 - |\Gamma_L|^2)(1 - |\Gamma_S|^2)}{|1 - \Gamma_L \Gamma_{out}|^2 \, |1 - S_{11} \Gamma_S|^2} \longrightarrow (9)$$

* Unilateral Power Gain (G_{TU})

The transducer power gain without considering feedback effect of an amplifier ie, $S_{12} = 0$, Then the gain is called unilateral power gain.

From equation (9),

$$G_{TU} = \frac{(1 - |\Gamma_L|^2) |S_{21}|^2 (1 - |\Gamma_S|^2)}{|1 - \Gamma_L S_{22}|^2 \, |1 - S_{11} \Gamma_S|^2}$$

STABILITY CONSIDERATIONS

The stability of an amplifier is an important considerations in microwave circuit design.

Types

1. Conditional Stability
2. Unconditional Stability

Conditional Stability

A network is conditionally stable if the real part of the input impedance (Z_{in}) and output impedance (Z_{out}) is greater than zero for some positive real source and load impedance at a specific frequency.

Unconditional Stability

A network is unconditionally stable if the real part of the input impedance (Z_{in}) and output impedance (Z_{out}) is greater than zero for all positive real source and load impedance at a specific frequency

STABILITY CIRCLES

Stability circle is a circle on smith chart that represents the boundary between those values of source and load impedance that cause instability and those do not.

Stability circles are a tool to analyze the stability of an amplifier or related circuits using a graphical technique.

Stability in RF means that the transistor is stable when embedded between 50Ω source and load, and it will not oscillate.

An amplifier circuit must be stable over the entire frequency range. The RF circuits (amplifier) tend to oscillate depending on an operating frequency termination (load).

Based on the value of reflection co-efficient (Γ), the circuit can be analyzed as,

* If $|\Gamma| > 1$ → then the magnitude of the return voltage wave increases called positive feedback - which causes instability

* If $|\Gamma| < 1 \rightarrow$ then return voltage wave is totally avoided, which is called as negative feedback.

* Two port network amplifier is characterized by its S-parameters. The amplifier is stable, when the magnitudes of reflection co-efficients are less than unity.

$$|\Gamma_L| < 1$$

and

$$|\Gamma_S| < 1$$

* Load reflection co-efficient (Γ_L) and source reflection co-efficient (Γ_S) are less than unity

Output Stability Circle

The output stability circle equation is,

$$\left[\Gamma_L^R - C_{out}^R\right]^2 + \left[\Gamma_L^I - C_{out}^I\right]^2 = r_{out}^2 \rightarrow (1)$$

circle radius, $r_{out} = \dfrac{|S_{12} S_{21}|}{|S_{22}|^2 - |\Delta|^2} \rightarrow (2)$

where,

$$\Delta = S_{11} S_{22} - S_{12} S_{21}$$

Center of stability circle, $C_{out} = C_{out}^{R} + j\,C_{out}^{I}$

$$= \frac{\left(S_{22} - S_{11}^{*}\Delta\right)^{*}}{|S_{11}|^{2} - |\Delta|^{2}} \rightarrow (3)$$

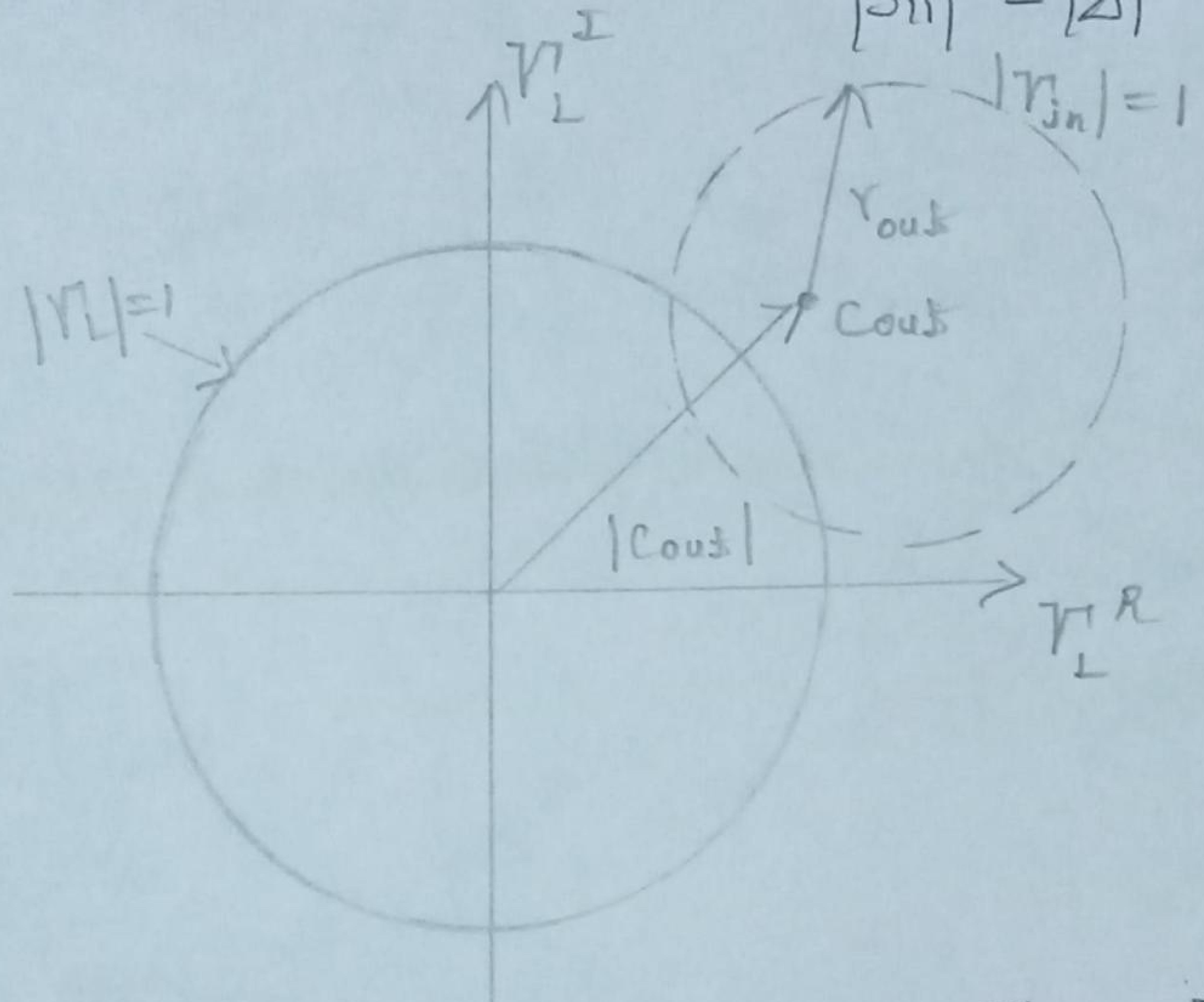

Fig: Output stability circle $\Gamma_{in} = 1$ in the Complex Γ_L plane

* When $\Gamma_L = 0$, then $|\Gamma_{in}| = |S_{11}|$, two stability domains of output stability circles are,

(i) For $|S_{11}| < 1 \rightarrow$ the origin [the point $\Gamma_L = 0$] is a part of stable region

(ii) For $|S_{11}| > 1 \rightarrow$ the origin [the point $\Gamma_L = 0$] part is unstable region. The only stable

Input stability circle

Input stability circle equation is,

$$\left[\Gamma_s^R - C_{in}^R\right]^2 + \left[\Gamma_s^I - C_{in}^I\right]^2 = r_{in}^2 \rightarrow (4)$$

Circle radius, $$r_{in} = \frac{|S_{12}S_{21}|}{\left|\,|S_{11}|^2 - |\Delta|^2\right|} \rightarrow (5)$$

Centre of input stability circle, $C_{in} = C_{in}^R + jC_{in}^I$

$$= \frac{(S_{11} - S_{22}^* \Delta)^*}{|S_{11}|^2 - |\Delta|^2} \rightarrow (6)$$

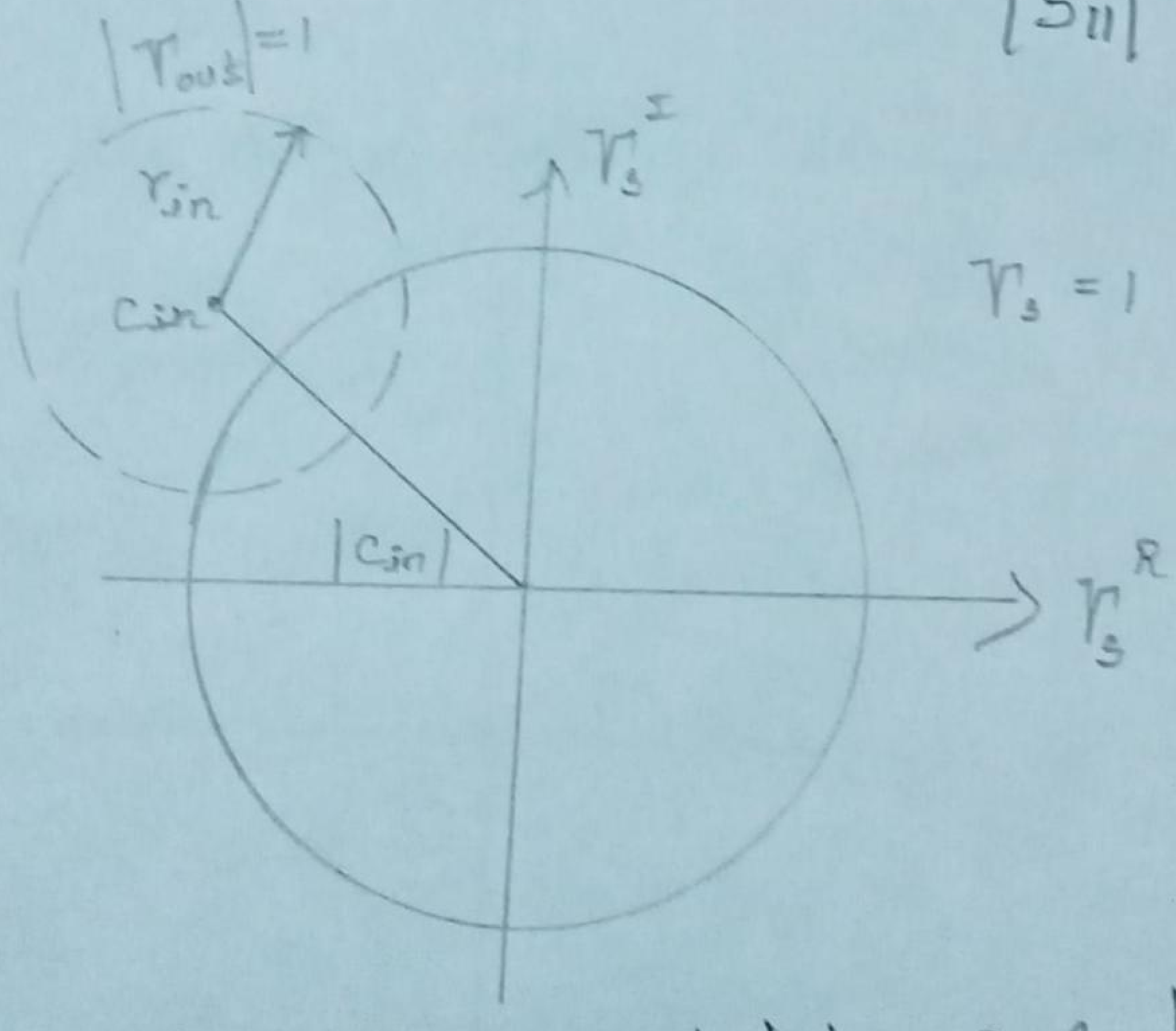

Fig: Input stability circle $|\Gamma_{out}| = 1$ in the complex Γ_s plane region is the shaded domain between the output stability circle $|\Gamma_{in}| = 1$ and $|\Gamma_L| = 1$ circle.

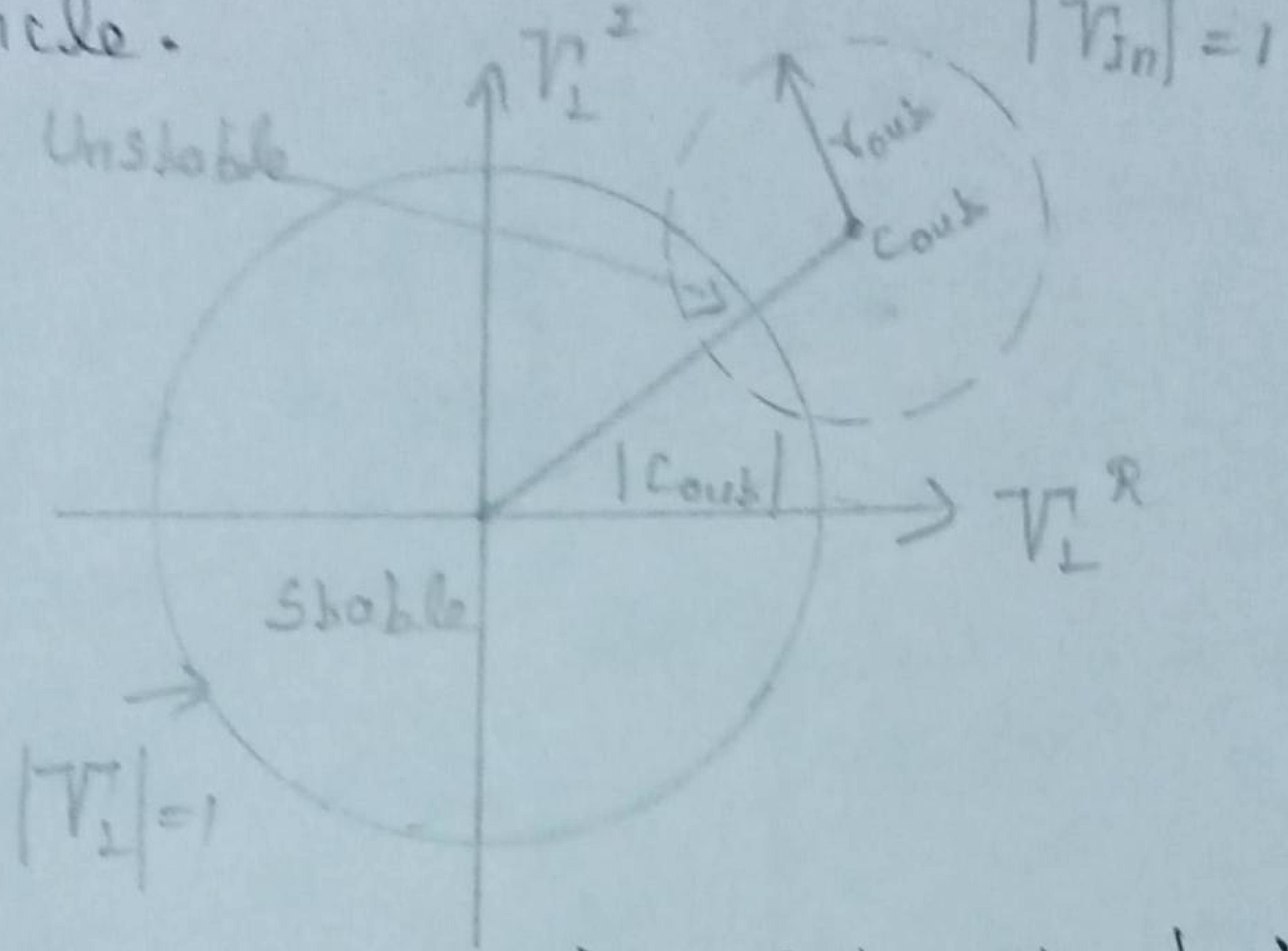

Fig: Output stability circle when $|S_{11}| < 1$

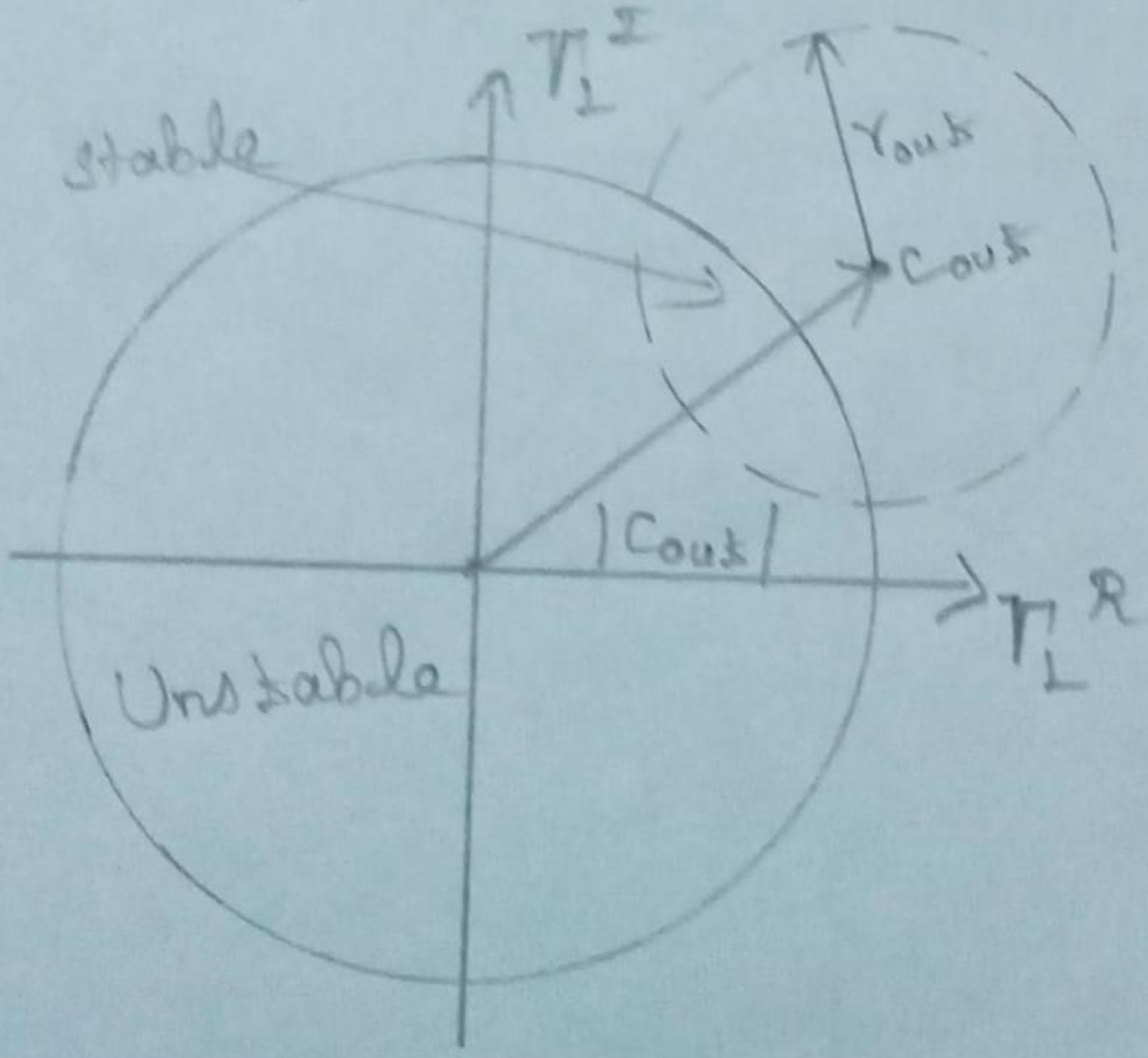

Fig: Output stability circle when $|S_{11}| > 1$

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* Two stability domains of input stability circles are,

(i) When $|S_{22}| < 1 \rightarrow$ the center ($\Gamma_s = 0$) must be stable

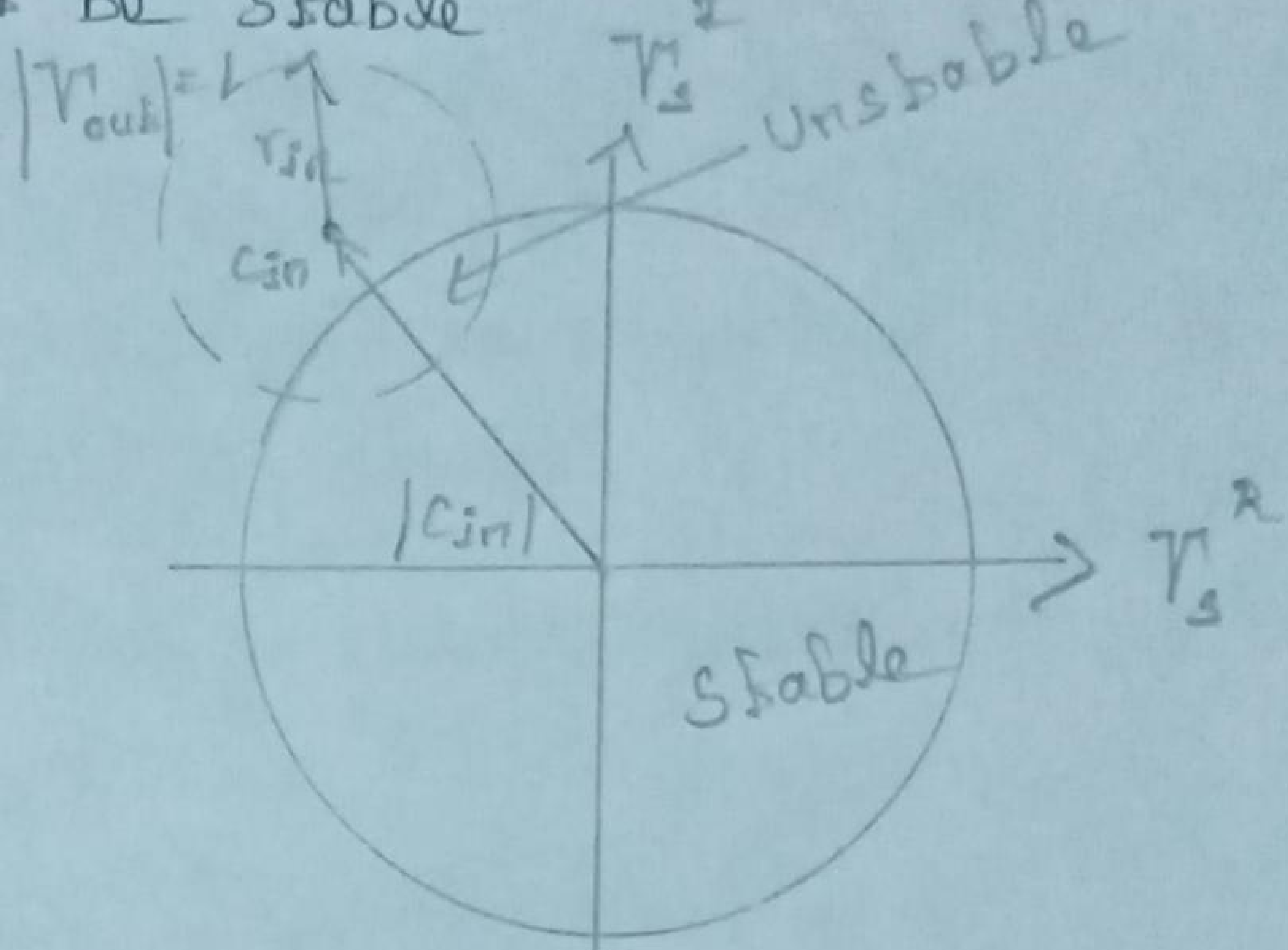

Fig: Input Stability Circle when $|S_{22}| < 1$

(ii) When $|S_{22}| > 1 \rightarrow$ the center ($\Gamma_s = 0$) becomes unstable

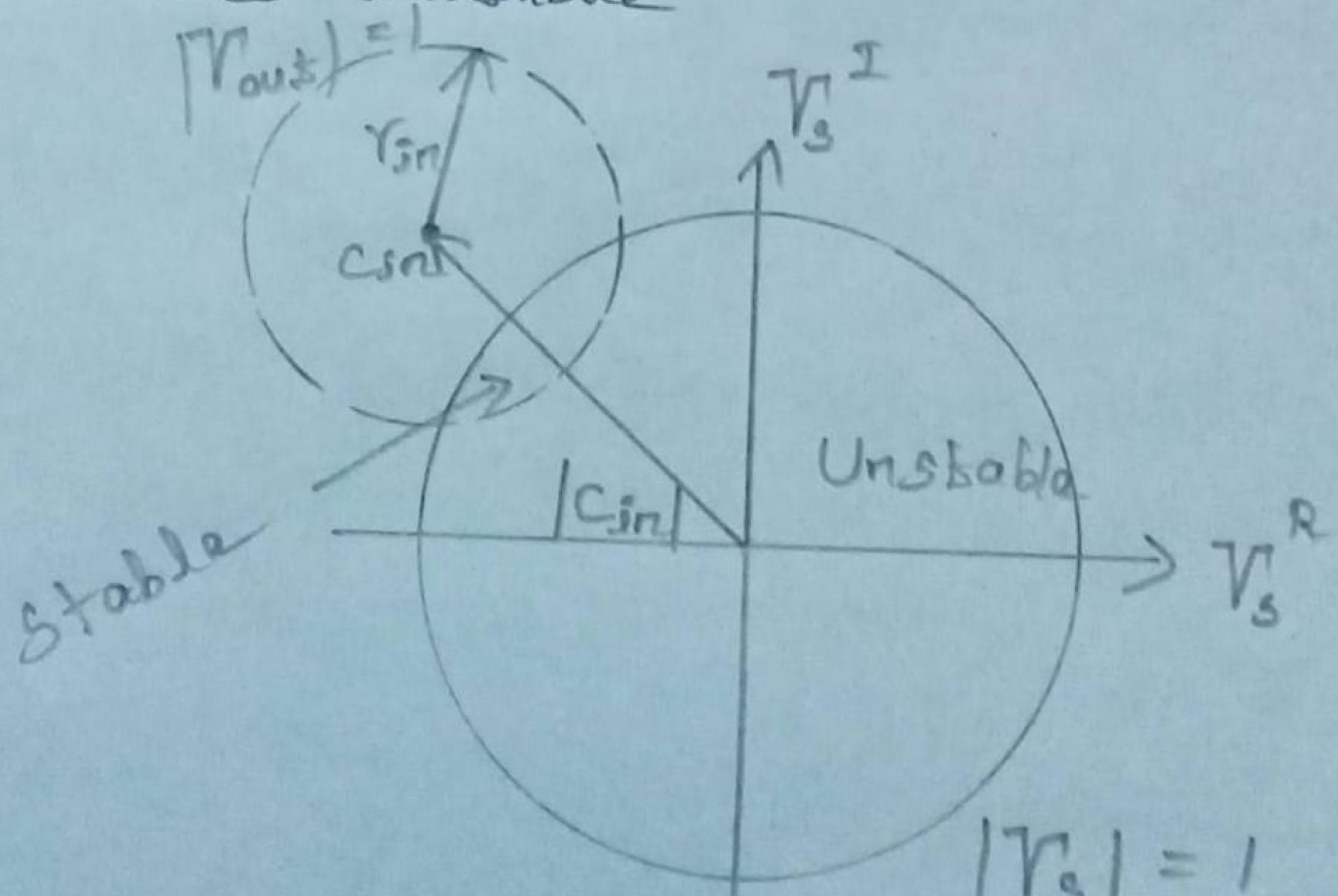

Fig: Input stability Circle when $|S_{22}| > 1$

Unconditional stability

An amplifier remains stable for any passive source and load at the selected frequency and bias conditions. Such a situation is referred to as an unconditional stability.

This situation is applicable to both input and output ports. For $|S_{11}| < 1$ and $|S_{22}| < 1$, the stability circles will be completely outside $|\Gamma_s| < 1$ and $|\Gamma_L| < 1$ circles.

The condition for stability in terms of stability factor k as,

$$k = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}||S_{21}|} > 1 \rightarrow (1)$$

The stability factor k is also referred as Rollet factor. It applies for both input and output ports.

Inequality $|\Delta| = |S_{11}S_{22} - S_{12}S_{21}| \le |S_{21}S_{22}| + |S_{12}S_{21}| \rightarrow (2)$

Equations (1) & (2) satisfies both the conditions for an unconditional stable design.

i.e., $|\Delta| < 1$

and $k > 1$